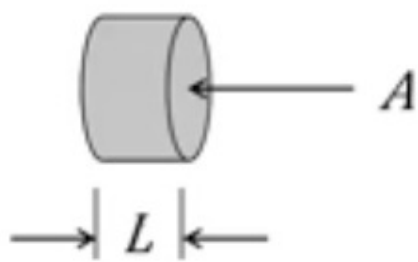
Learning Objectives

Electrical Conductivity and the Drude Model

- Distinguish between extrinsic (I, V, R) and intrinsic (J, E, σ, ρ) electrical properties
- Apply the Drude model to describe electron behavior as an ideal gas in metals
- Calculate key parameters: drift velocity, electron mobility, mean free path, and relaxation time
- Derive the relationship between conductivity, carrier concentration, and mobility ($\sigma = ne\mu$)
- Estimate conductivity parameters for simple metals using the Drude model
- Recognize the limitations and failures of the Drude model in predicting temperature dependence and trends across different metals

Ohm's Law

Ohm's Law (Extrinsic) $$I = \frac{V}{R}$$



Current density, $J = I/A$

Electric field intensity, $E = V/L$

Resistivity, $\rho = RA/L$

} Intrinsic Properties

Conductivity

Ohm's Law (Extrinsic) $I = \frac{V}{R}$

Ohm's Law (Intrinsic)

$$I = \frac{V}{R} \longrightarrow JA = \frac{(EL)}{(L\rho/A)} = \frac{EA}{\rho}$$

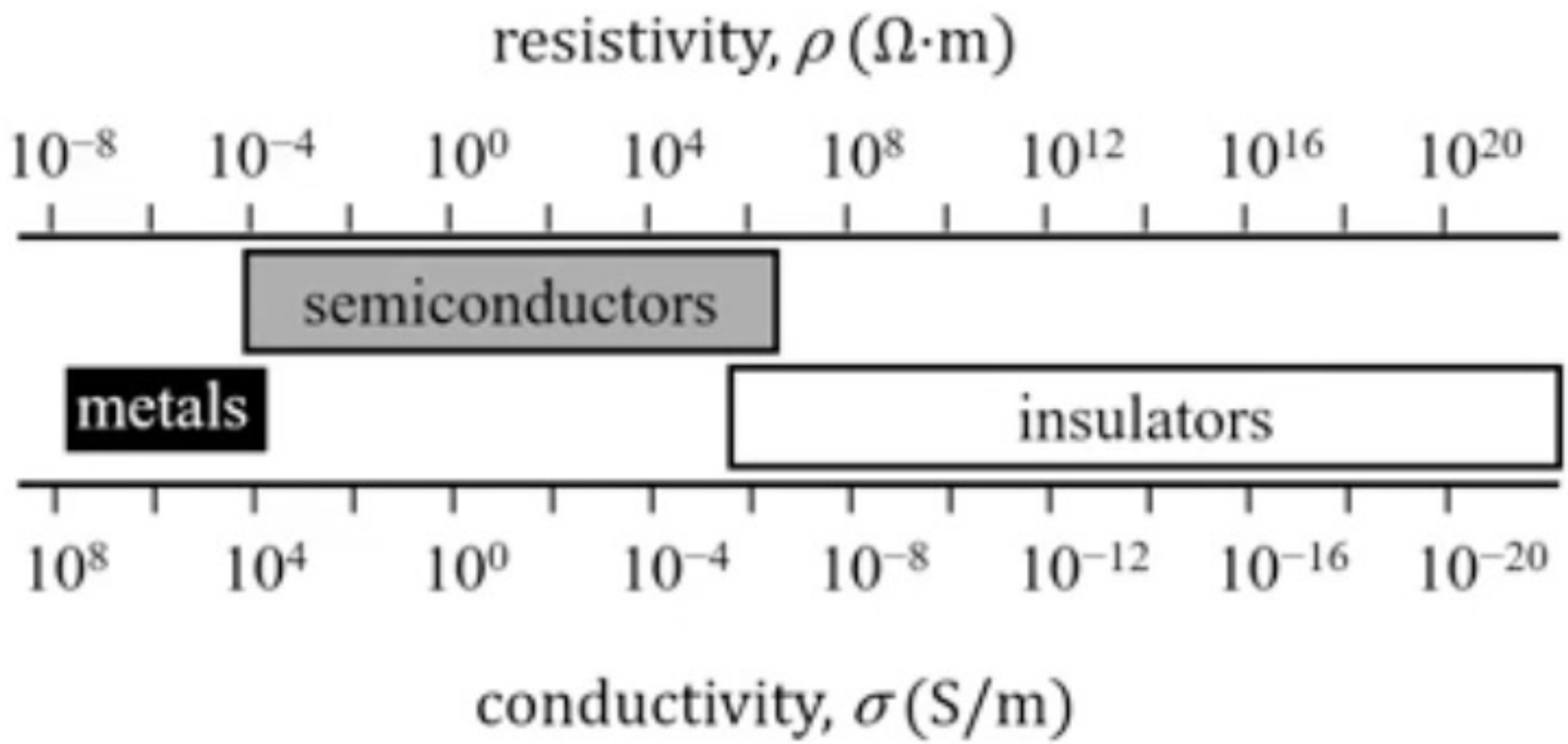
$$J = \frac{E}{\rho}$$

Resistivity, ρ ($\Omega{\cdot}m$)

$$J = \sigma E$$

Conductivity, $\sigma = 1/\rho$ ($\Omega^{-1}m^{-1}$ = S/m)

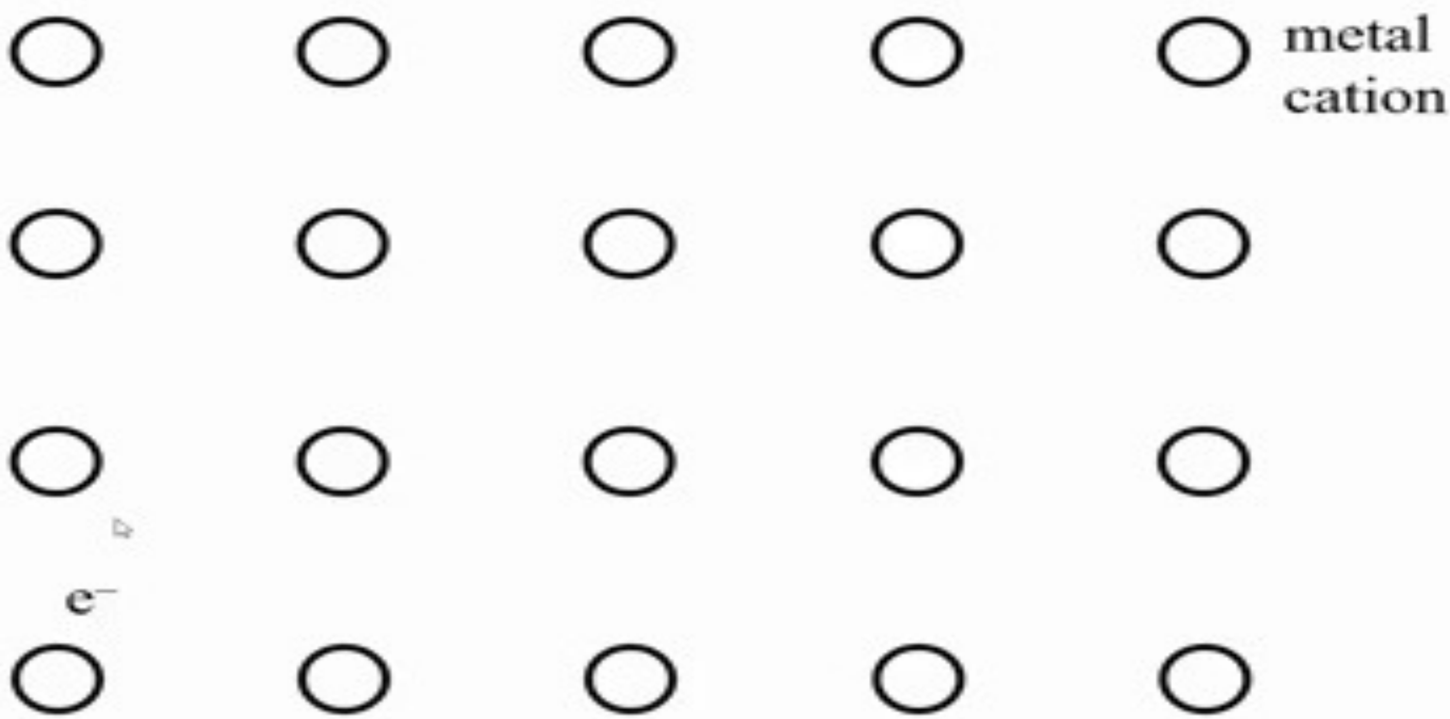
Conductivity of Materials



Conductivity of Select Materials

Substance	σ (S/m)	*Substance*	σ (S/m)
Ag	6.2×10^7	$Bi_2Ru_2O_7$	2×10^5
Cu	5.9×10^7	$LaNiO_3$	1×10^5
Al	3.8×10^7	doped polyacetylene	8×10^4
Na	2.1×10^7	Fe_3O_4	2×10^4
ReO_3	1.1×10^7	$YBa_2Cu_3O_7$*	1×10^2
Ti	2.5×10^6	Ge	2×10^0
La	1.6×10^6	Si	10^{-3}
$SrMoO_3$	1.0×10^6	NiO	10^{-8}
Bi	7.7×10^5	Al_2O_3	10^{-12}
Mn	6.2×10^5	S	10^{-15}
NbN	4×10^5	SiO_2 (Quartz)	10^{-16}
TiO	3×10^5	Teflon	10^{-22}

Drude Model (Free electron gas)



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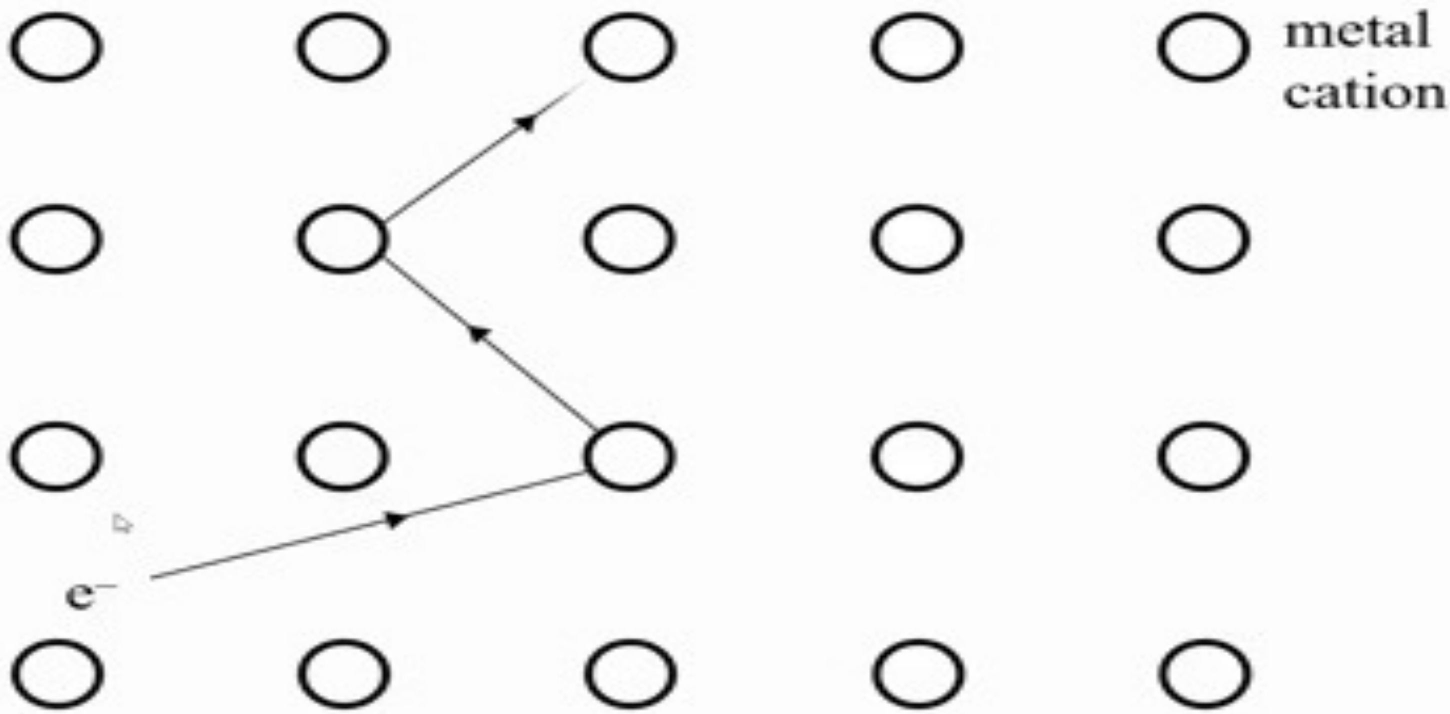
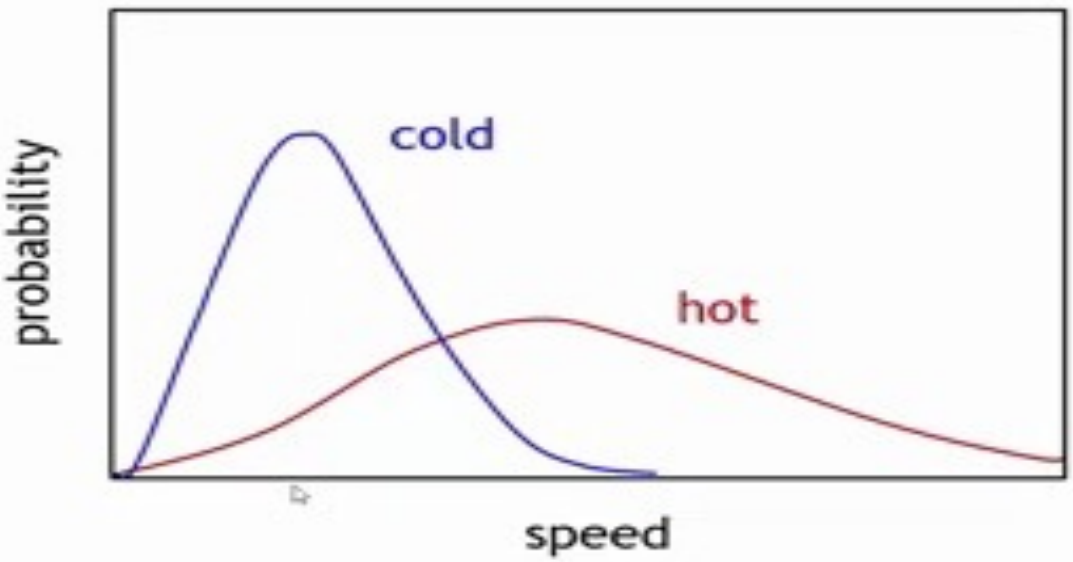
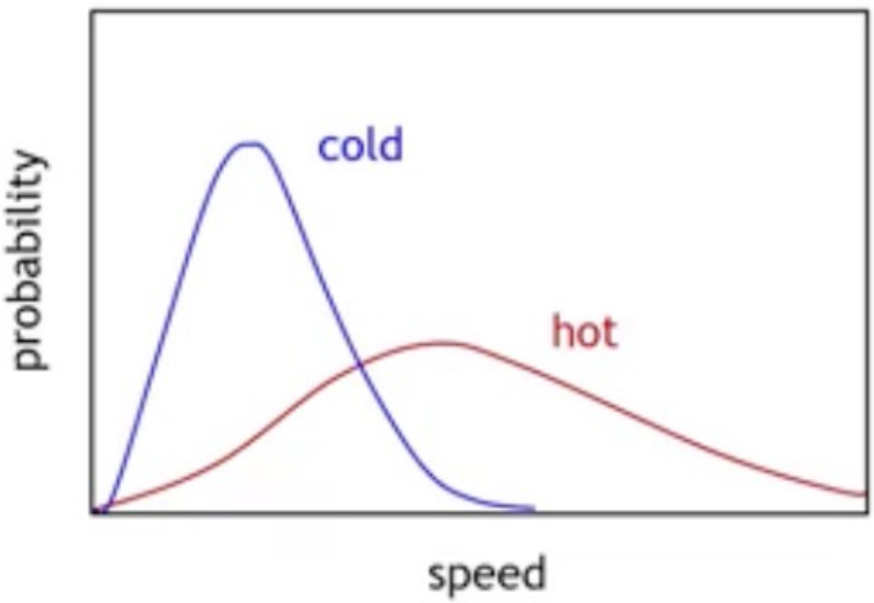


Boltzmann distribution



This is the distribution of electron energies and velocities used in the Drude model

Boltzmann distribution



This is the distribution of electron energies and velocities used in the Drude model

Average electron energy

$$E = \frac{3}{2}kT$$

Root mean square electron velocity

$$v_{rms} = \sqrt{\frac{2E}{m_e}} = \sqrt{\frac{3kT}{m_e}}$$

$v_{rms} \approx 10^5$ m/s
at $T = 300$ K

Mean Free Path, Relaxation Time

Relaxation time
$\tau = 1 \times 10^{-14}$ s

Mean free path
$\ell \approx 1$ nm $= 1 \times 10^{-9}$ m

$$\tau = \frac{\ell}{v}$$

Velocity
$v = 1 \times 10^{5}$ m/s

Drift Velocity

The electrons are moving very fast, but randomly. To get a net flow of current in one direction we must apply an electric field. The net speed of the electrons being pushed by the electric field is called drift velocity.

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The drift velocity increases as the electric field gets larger or the relaxation time gets larger ($v_d \cong 0.2$ m/s)

Electron Mobility

Electron mobility, μ is a measure of how easily the electrons move in response to an applied field.

$$\mu = \frac{v_d}{E}$$

$$\mu = \left(\frac{eE\tau}{m_e}\right)\frac{1}{E}$$

$$\mu = \frac{e\tau}{m_e}$$

Mobility increases as the relaxation time increases.

Conductivity

$$J = nev_d$$

n = concentration of electrons (m^{-3})

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$$J = ne\mu E = (ne\mu)E$$

$$J = \sigma E$$

$$\sigma = ne\mu$$

Conductivity increases as the concentration of mobile electrons (carriers) and/or their mobility increases

Adding up the Numbers, Sodium

Quantity	Method of determination	Value
Conductivity, σ	measured	$2.1 \times 10^{7}\ \Omega^{-1}\ m^{-1}$
e^- concentration, n	from crystal structure	$2.5 \times 10^{28}\ m^{-3}$
Mobility, μ	$\mu = \sigma / ne$	$5.2 \times 10^{-3}\ m^2/V\cdot s$
Relaxation time, τ	$\tau = \mu m_e / e$	3×10^{-14} s
Rms Velocity, v_{rms}	$v_{rms} = \sqrt{3kT/m_e}$	1×10^{5} m/s
Mean free path, ℓ	$\ell = v_{rms}\tau$	3×10^{-9} m

Valence electron concentration and conductivity

Table 10.2 Conductivity σ does not scale with the valence-electron concentration n.

Metal	n (m^{-3})	σ (S/m)	Metal	n (m^{-3})	σ (S/m)
Rb	1.1×10^{28}	0.8×10^{7}	Ag*	5.9×10^{28}	6.2×10^{7}
K	1.3×10^{28}	1.4×10^{7}	Au*	5.9×10^{28}	4.5×10^{7}
Na	2.5×10^{28}	2.1×10^{7}	Cu*	8.4×10^{28}	5.9×10^{7}
Ca	4.6×10^{28}	2.9×10^{7}	Zn*	13.1×10^{28}	1.7×10^{7}
Mg	8.6×10^{28}	2.3×10^{7}	Sc	12.0×10^{28}	0.18×10^{7}
Al	18.0×10^{28}	3.8×10^{7}	Ti	20.5×10^{28}	0.25×10^{7}

* The d subshells of Cu, Ag, Au, and Zn are assumed to be full and are not counted in the valence-electron count.

Why doesn't σ scale with valence electron concentration, n, once we move beyond the alkali metals?

3. The lecture describes electrons having a very high random thermal velocity (~100,000 m/s) and a much slower drift velocity (~0.2 m/s). Which of these is directly responsible for the net flow of current (J)?

A. Neither; current is caused by the electric field itself, not electron motion.

B. The drift velocity, which is the net motion in response to the electric field.

C. Both contribute equally to the current.

B. The drift velocity, which is the net motion in response to the electric field.

✓ **That's right!**

This slow, net movement in one direction, superimposed on the random thermal motion, is what constitutes the electrical current.

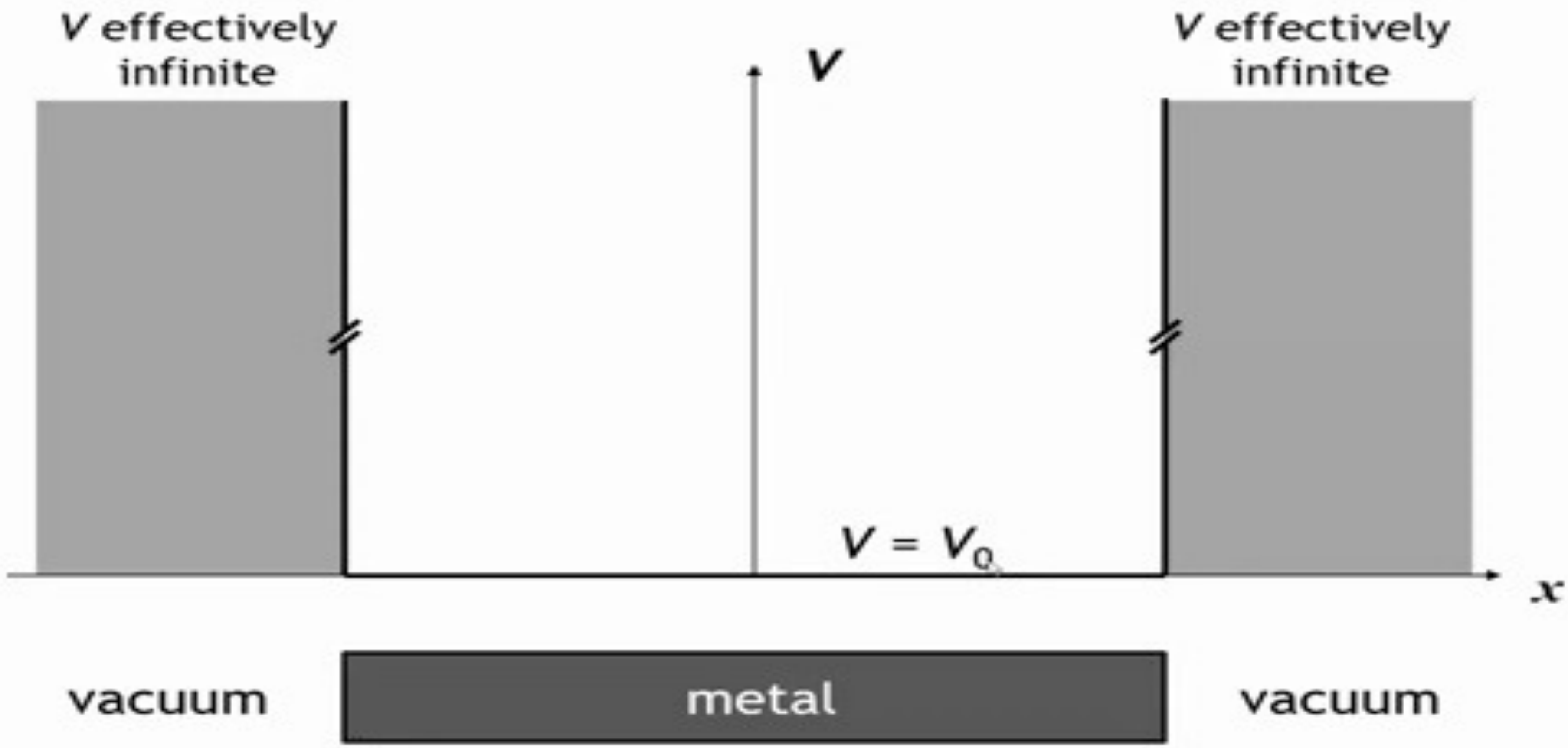
Homework:
10.1-10.4, 10.8, 10.9, 10.10

Learning Objectives

Quantum Mechanics and Metallic Conductivity

- Explain the transition from the Drude model to the free electron model and identify the key quantum mechanical principles incorporated in the free electron model
- Describe the parabolic E(k) relationship in the free electron model and relate it to the particle-in-a-box treatment
- Connect effective mass (m^*) to band curvature and predict how band structure affects carrier mobility and conductivity
- Apply the Fermi-Dirac distribution to explain which electrons contribute to electrical conductivity and why only states near the Fermi level matter
- Explain the temperature dependence of metallic conductivity, including the role of phonon scattering and residual resistivity
- Predict relative conductivities across the periodic table by analyzing band structure, particularly distinguishing between d-band and s-band contributions to transport
- Justify why coinage metals (Cu, Ag, Au) exhibit exceptionally high conductivities compared to transition metals based on their electronic band structures

Potential used for free electron model



Free Electron Model

Crystal orbital wavefunction
(Bloch function)

$$\psi(x) = e^{ikx}u(x)$$

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Time independent Schrödinger eq'n

$$\mathcal{H}\psi(x) = E\psi(x)$$

$$E = V_o + \frac{\hbar^2}{2m_e}k^2$$

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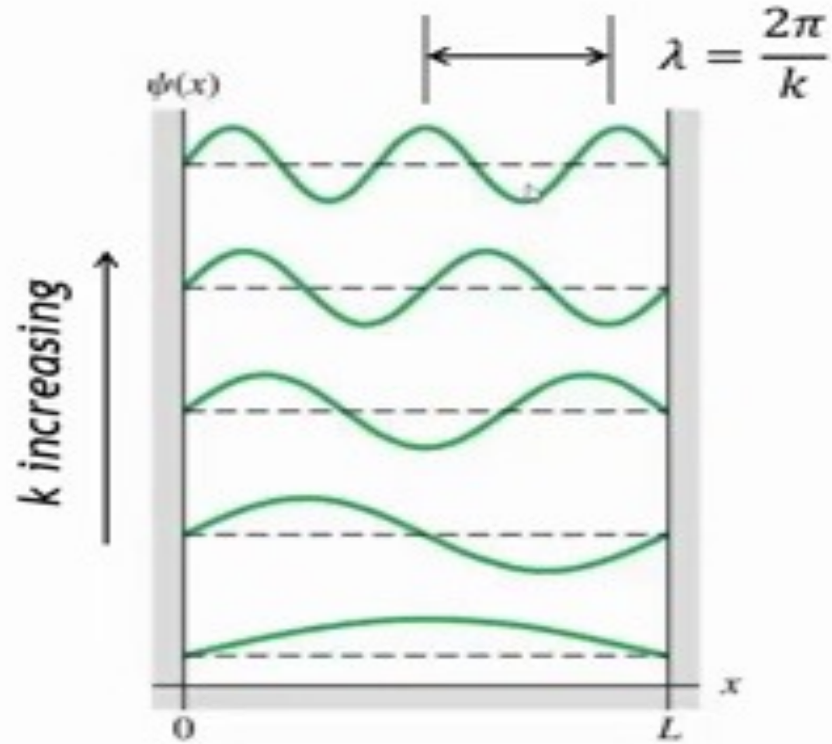
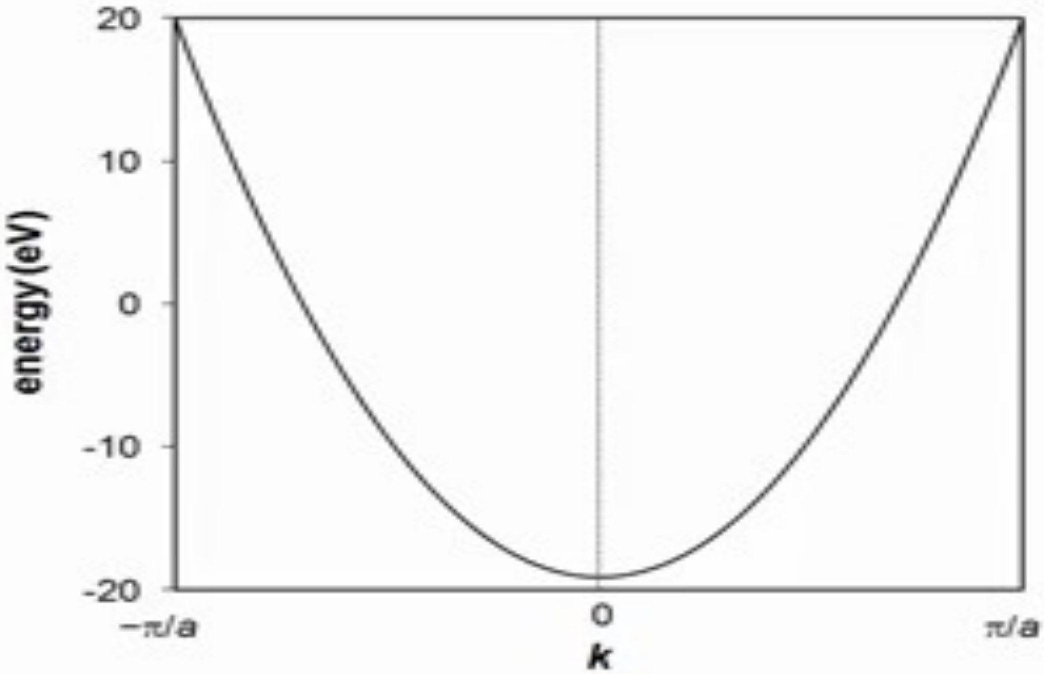


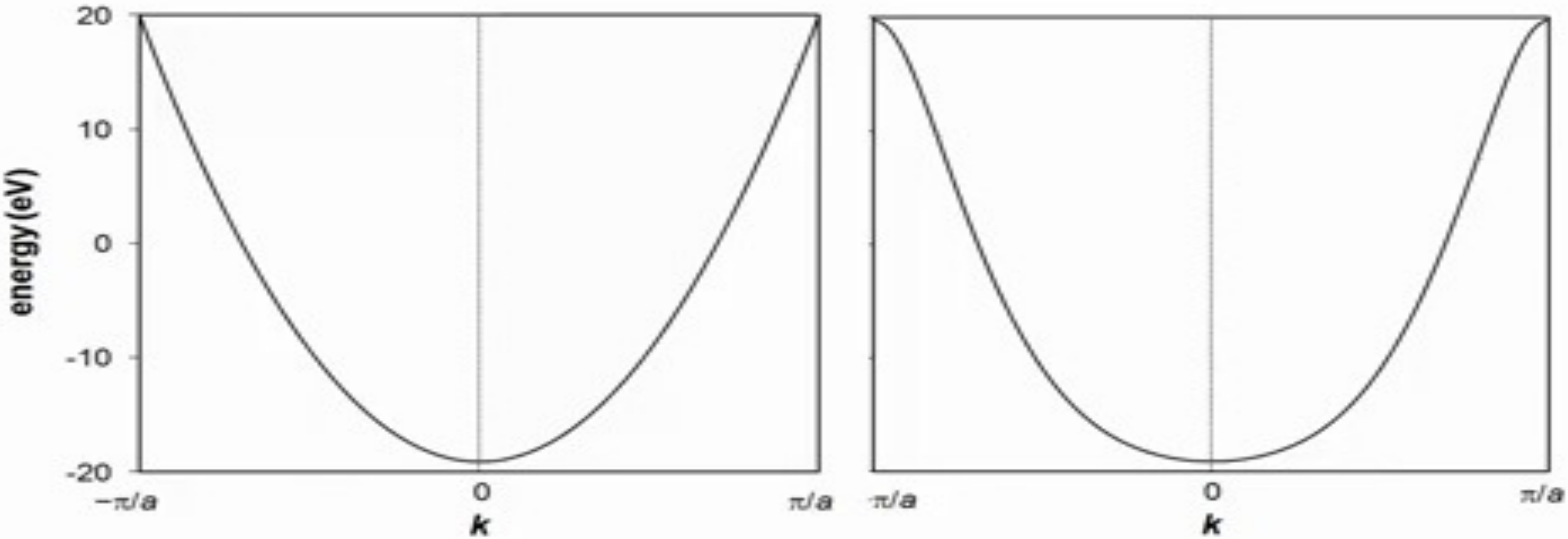
Band structure



$$E = V_o + \frac{\hbar^2}{2m^*}k^2$$

effective mass, m^*

Band structure comparison



free-electron model

Tight binding model
H atom chain

Effective mass

$$E = V_o + \frac{\hbar^2}{2m^*} k^2$$

Differentiating twice with respect to *k* we get the effective mass

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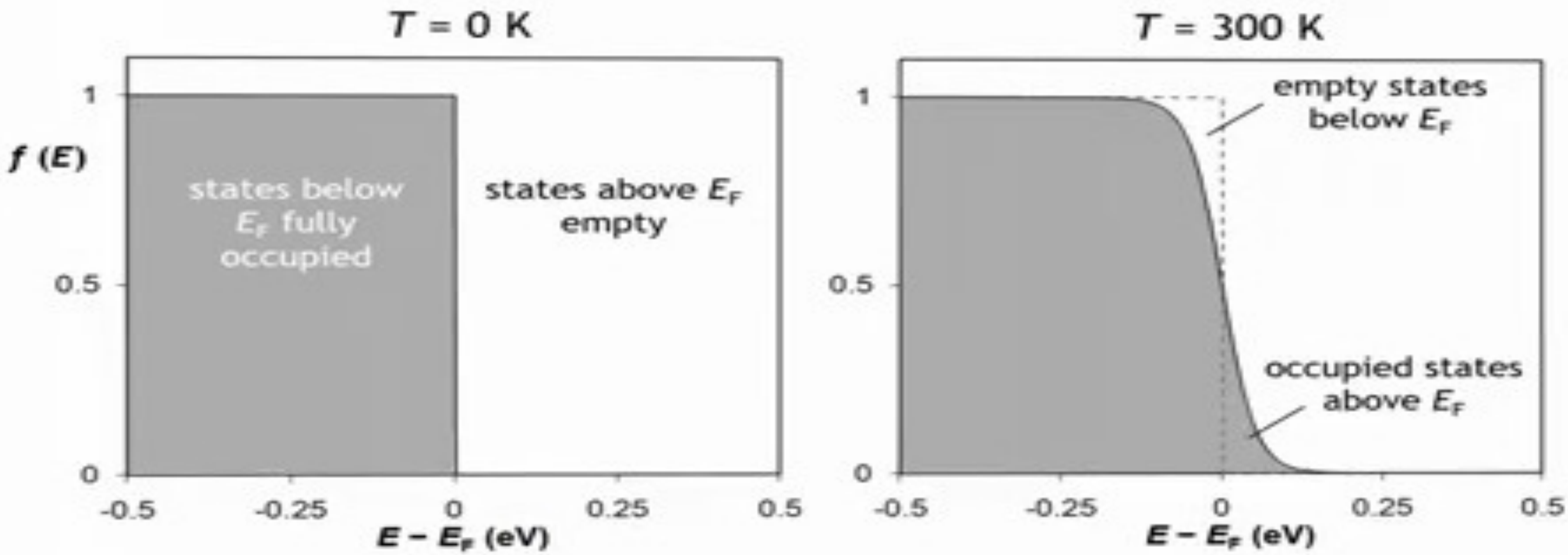
$$\mu = \frac{e\tau}{m^*}$$

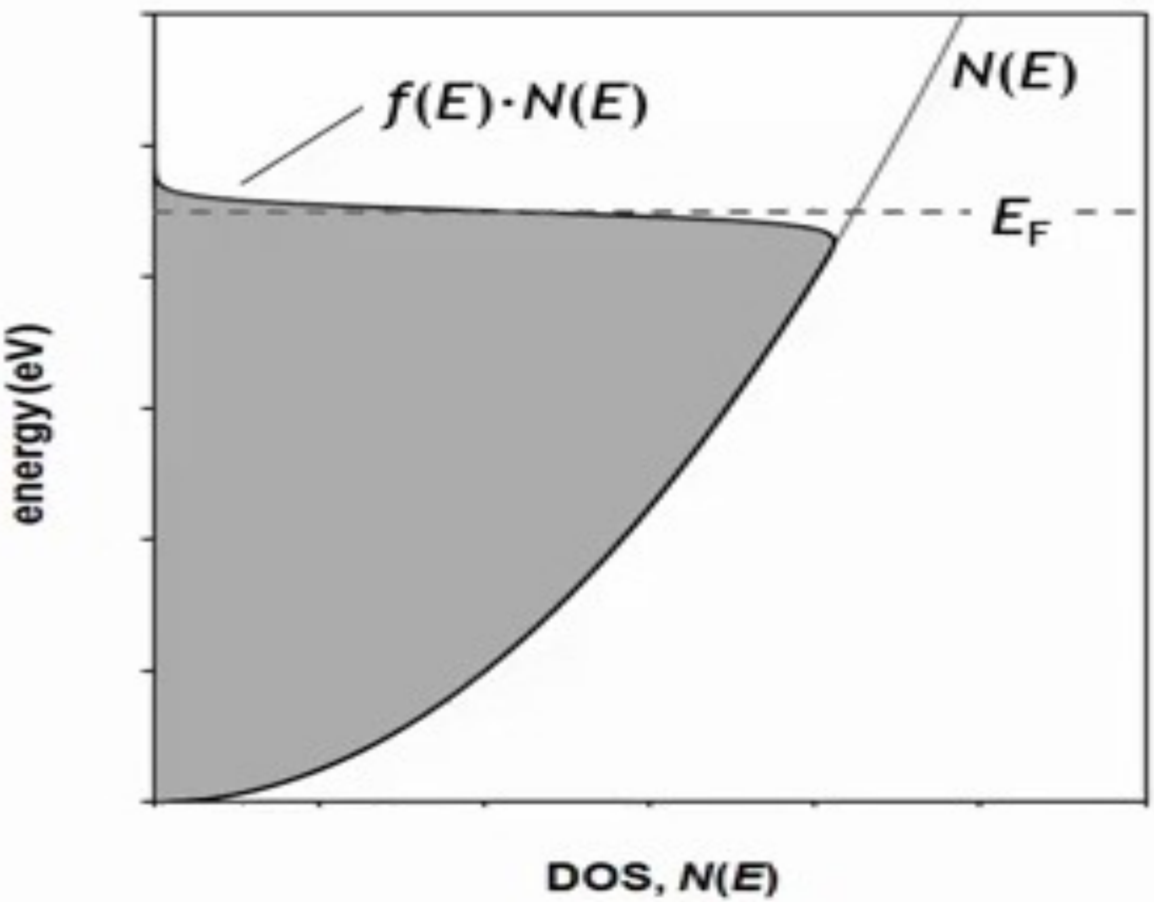
Small effective mass (m^*) → High mobility (μ)

Fermi-Dirac distribution



$$f(E) = \frac{1}{1 + \exp[(E - E_F)/k_B T]}$$

Density of States - Free electron model

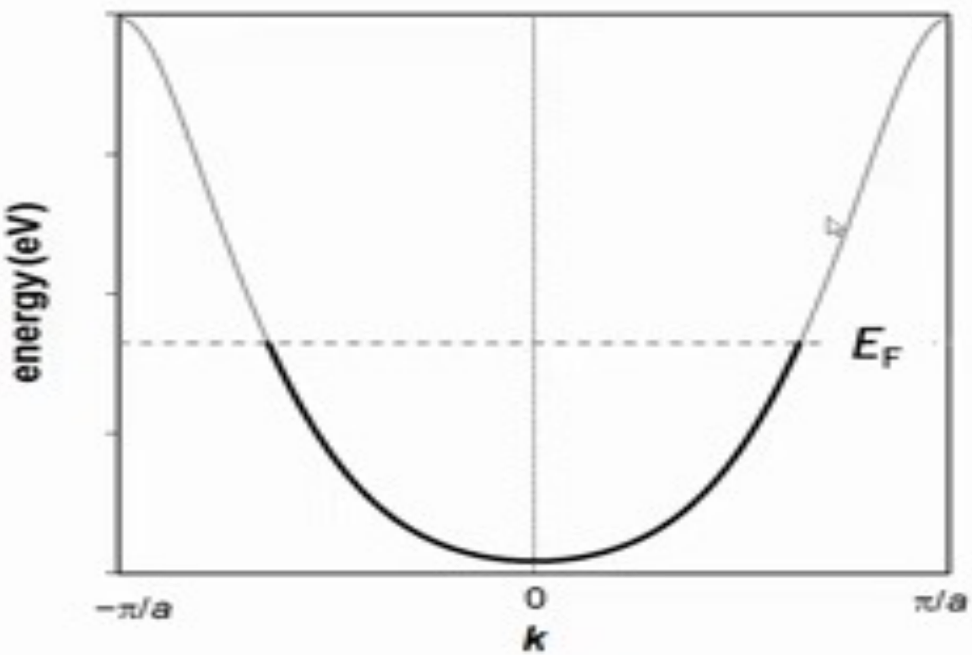


Only a fraction of the valence electrons (those near the Fermi level) carry the current.

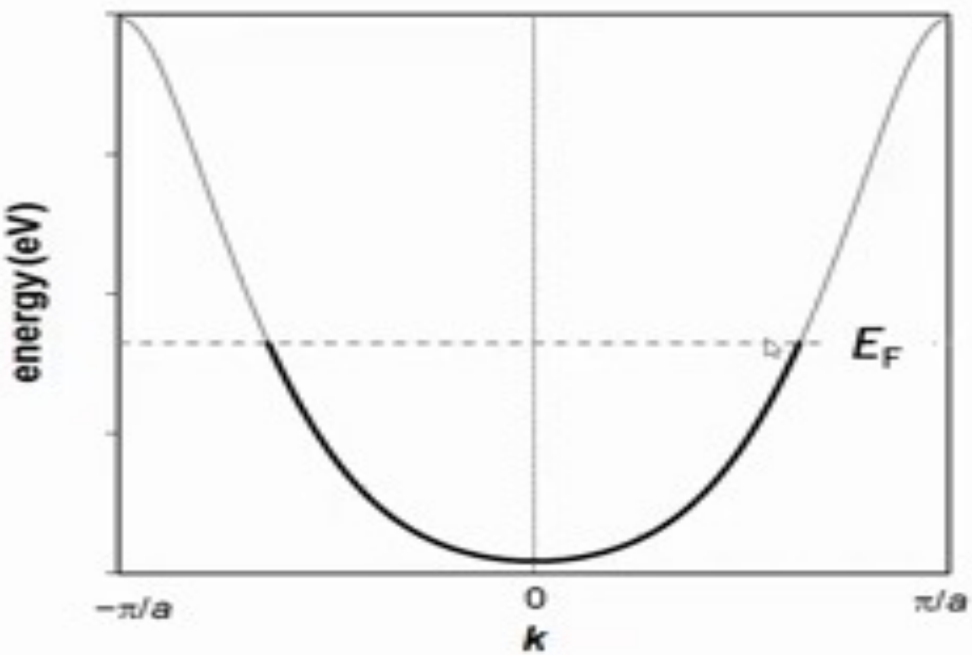
$$N(E) \propto E^{1/2}$$

As the band filling increases the DOS at E_F increases.

Fermi Velocity



Fermi Velocity



$$p = \hbar k$$

$$\downarrow$$

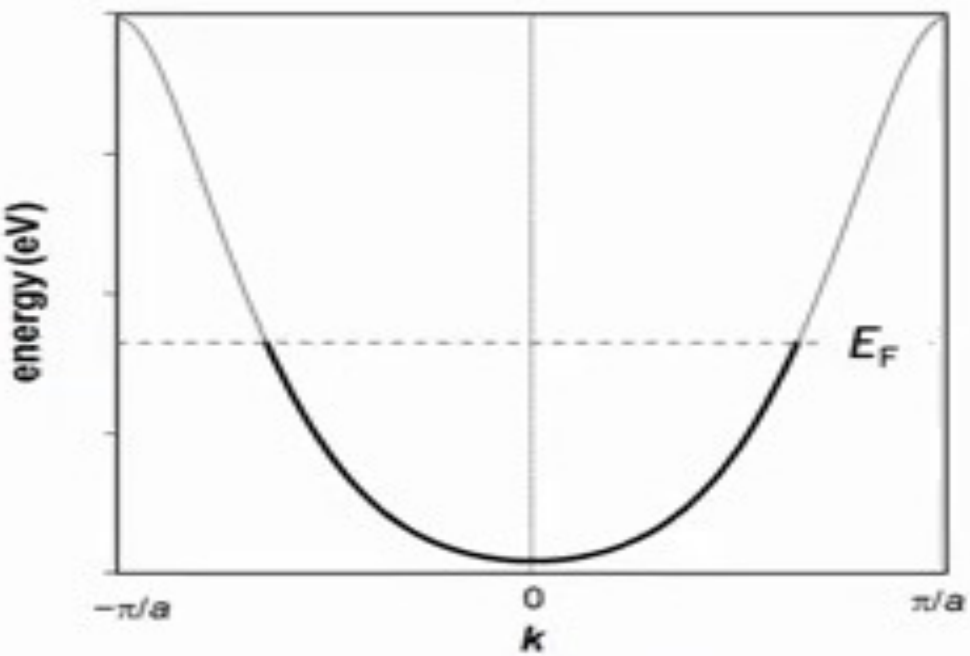
$$m^* v = \hbar k$$

$$\downarrow$$

$$v = \frac{\hbar k}{m^*}$$

We can estimate the velocity of the electrons at the Fermi level from their energy

Fermi Velocity



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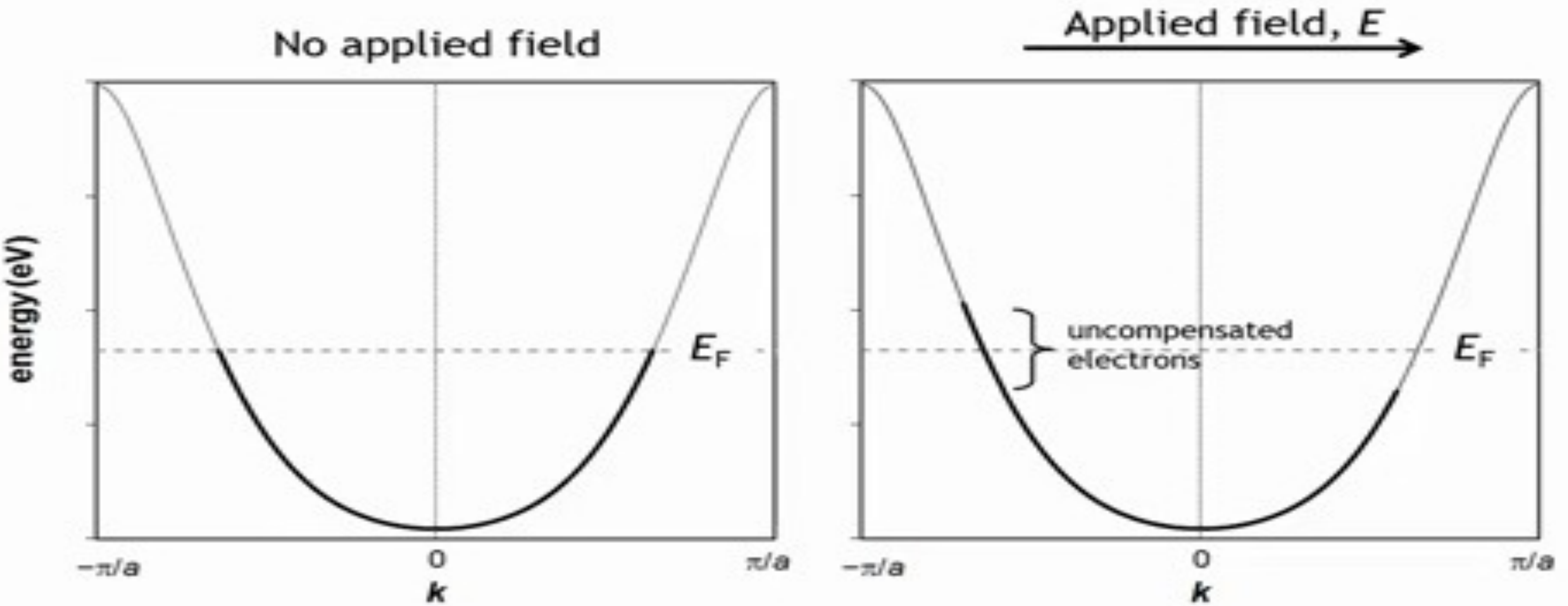
We can estimate the velocity of the electrons at the Fermi level from their energy

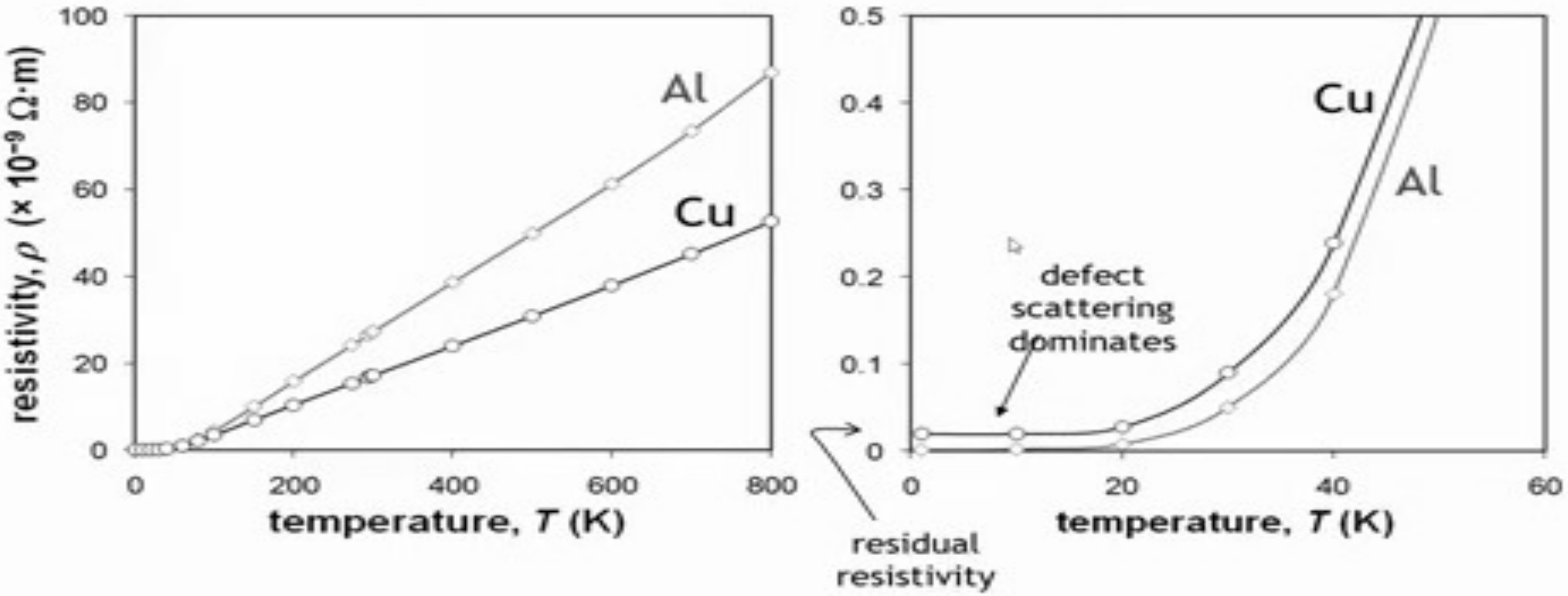
$$E_F = \frac{1}{2} m^* (v_F)^2 \longrightarrow v_F = \sqrt{\frac{2E_F}{m^*}}$$



$$v_F = \sqrt{\frac{2E_F}{m^*}}$$

This is the velocity of electrons at the Fermi level in the free electron model. For Cu E_F = 7.0 eV and $v_F = 1.6\times10^6$ m/s, much faster than our Drude model estimate.

Temperature dependence (metal)



Electron mean free path in high puritity Al
29 nm at 300 K, 0.7 mm at 1 K!

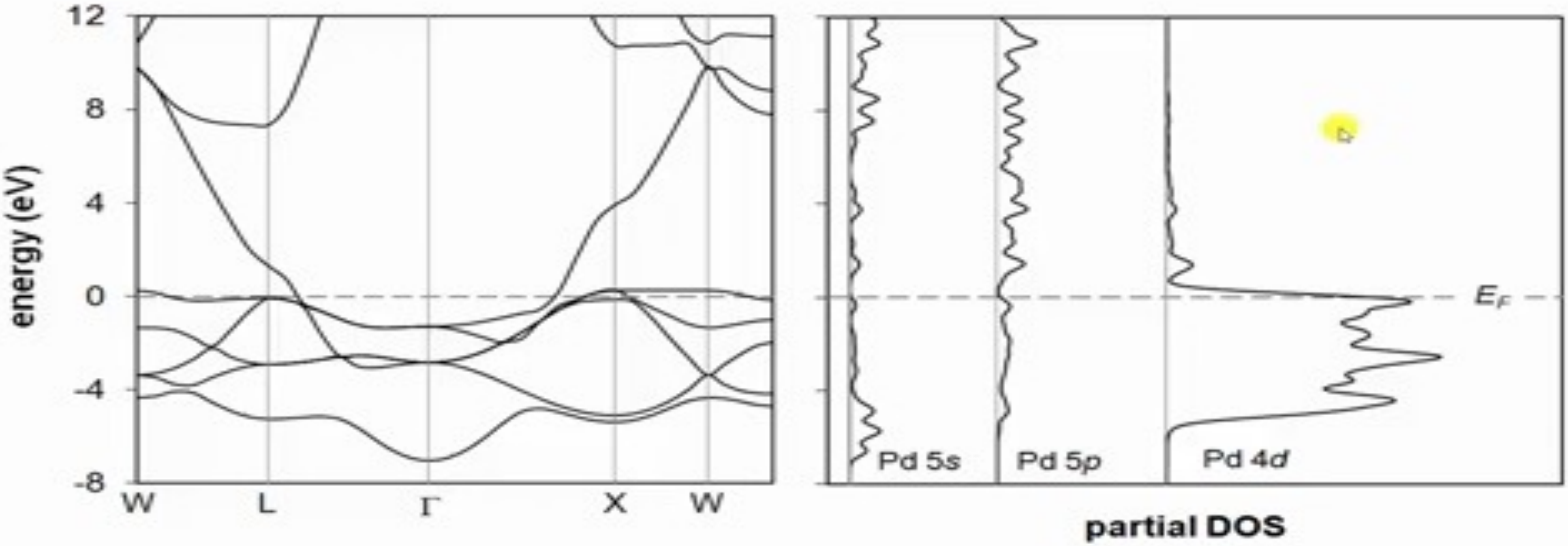
Conductivities of Transition Metals

3	4	5	6	7	8	9	10	11
Sc 0.18 *hcp***	**Ti** 0.25 *hcp*	**V** 0.50 *bcc**	**Cr** 0.79 *bcc*	**Mn** 0.06 *other*	**Fe** 1.0 *bcc*	**Co** 1.7 *hcp*	**Ni** 1.4 *fcc**	**Cu** 5.9 *fcc*
Y 0.18 *hcp*	**Zr** 0.24 *hcp*	**Nb** 0.67 *bcc*	**Mo** 2.0 *bcc*	**Tc** 0.50 *hcp*	**Ru** 1.4 *hcp*	**Rh** 2.3 *fcc*	**Pd** 1.0 *fcc*	**Ag** 6.2 *fcc*
Lu 0.18 *hcp*	**Hf** 0.53 *hcp*	**Ta** 0.77 *bcc*	**W** 2.0 *bcc*	**Re** 0.56 *hcp*	**Os** 1.2 *hcp*	**Ir** 2.1 *fcc*	**Pt** 0.94 *fcc*	**Au** 4.5 *fcc*

Conductivities, σ (× 10^7) in S/m, of the transition metals. By comparison Na is 2.1, Mg 2.3 and Al is 3.8.

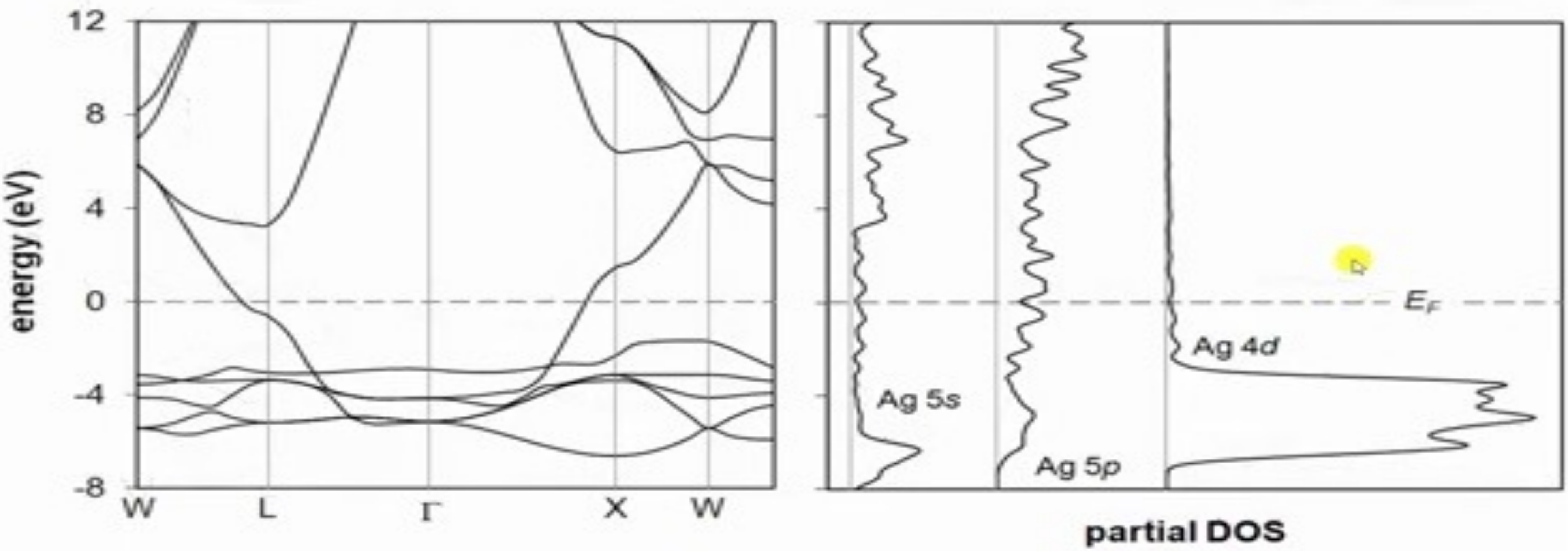
What is responsible for the large jump between groups 10 and 11?

Band structure palladium



Fermi level cuts through 4d bands. These bands are fairly narrow, which translates to higher effective mass and lower mobility.

Band structure silver



Fermi level cuts through 5s bands. This band is wide, which translates to lower effective mass and higher mobility.

Homework:
10.5-10.7

Go to gradescope quiz.