CHEM 548: MATERIALS CHEMISTRY

POINT SYMMETRY, POINT GROUPS, AND CRYSTAL SYSTEMS

Meeting 2 · 08/26/2026

Reading: Woodward, Karen, Evans, and Vogt, *Solid State Materials Chemistry* Ch. 1 §1.1.1, §§1.1.2–1.1.3

LEARNING OBJECTIVES

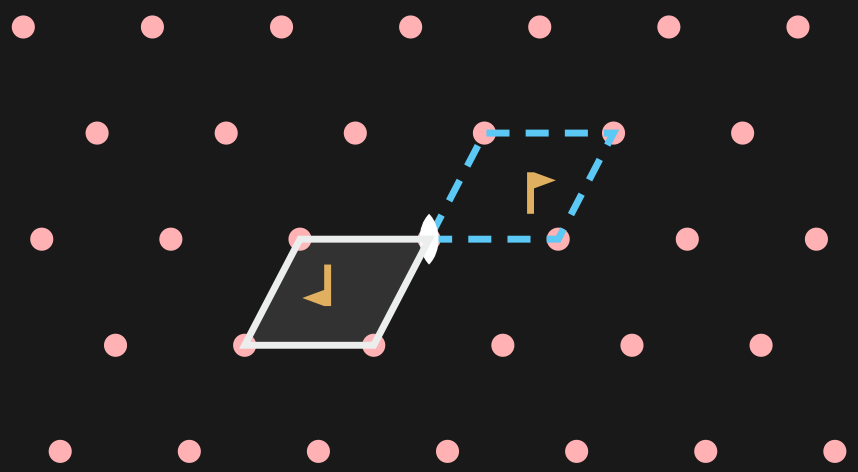
Five elements	define and describe identity, inversion, mirror, rotation, and rotoinversion
Point group	say what makes a set of operations one, and assemble it by closing the set
Seven crystal systems	identify each by the minimum symmetry it requires
Exactly 32	account for why there are 32 point groups, and connect them to their crystal system
Directions and planes	name a crystallographic direction and a lattice plane inside a unit cell

OBLIQUE: 1 ↔ 2

a ≠ b, γ arbitrary. In lecture 1 we chose the cell that shows each lattice's symmetry; now we write that symmetry down as a Hermann–Mauguin symbol, a list of the point symmetry elements the pattern keeps. The bare lattice carries each shape's **maximum** symmetry; a motif can only lower it. Each title reads minimum ↔ maximum.

Lattice alone: the maximum

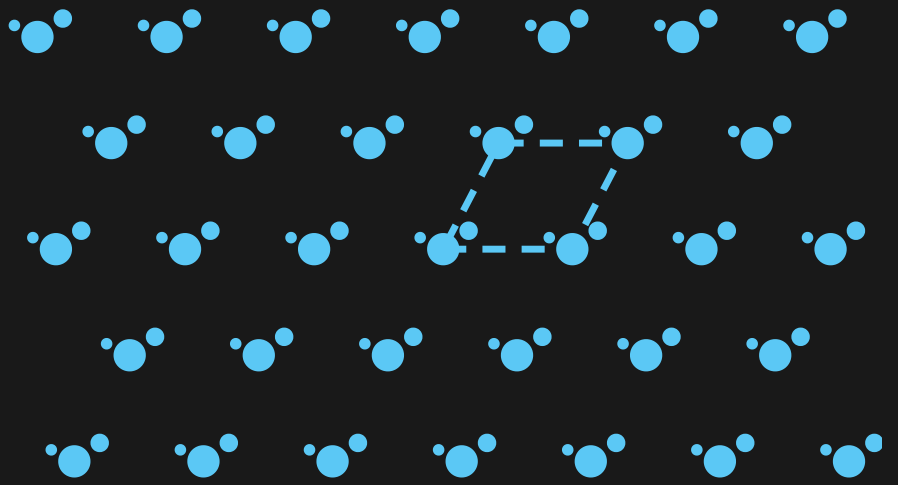
Every lattice keeps a twofold rotation



Hermann–Mauguin: **2**

With a motif: down to the minimum

An asymmetric molecule removes it



Hermann–Mauguin: **1**

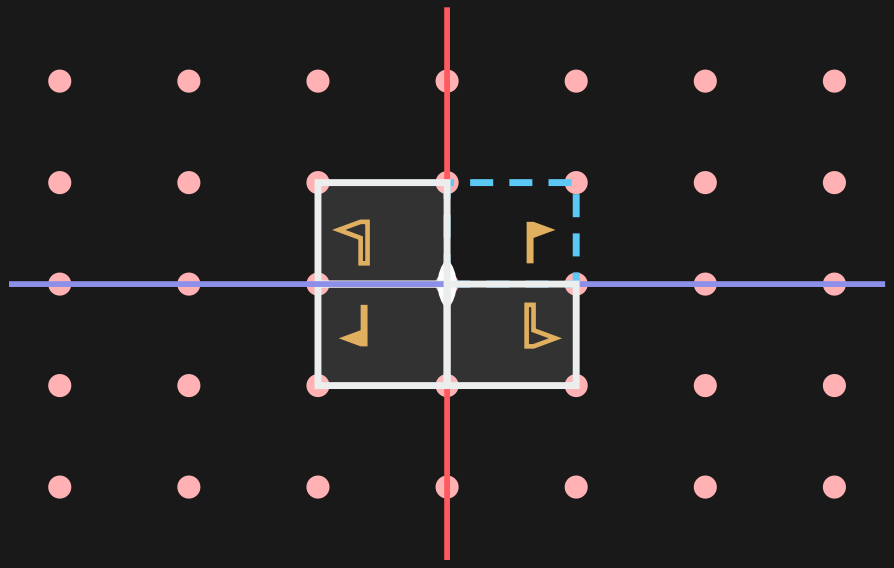
Per *Solid State Materials Chemistry*, §§1.1.1–1.1.2.

RECTANGULAR: m ↔ 2mm

$a \neq b$, $\gamma = 90°$

Lattice alone: the maximum

Twofold plus two perpendicular mirror families

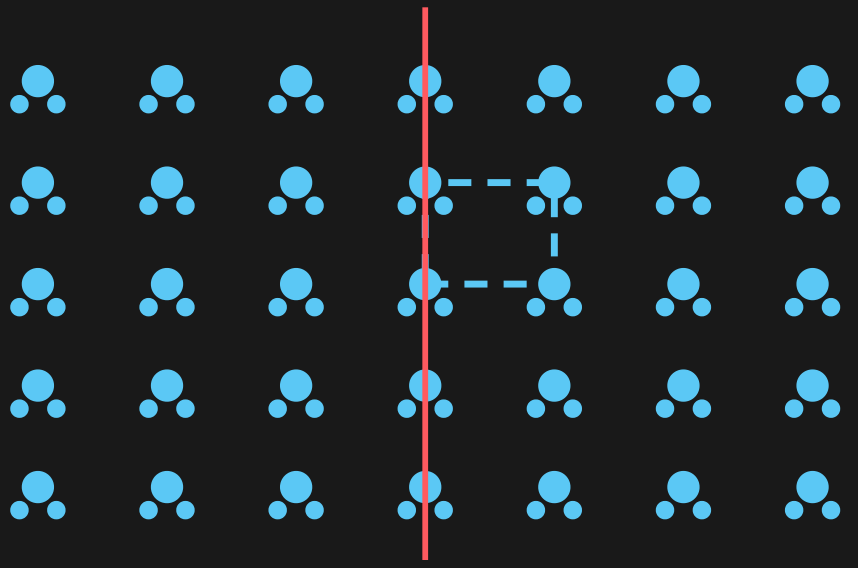


Hermann–Mauguin: **2mm**

The fold flips the flag: mirrors reverse handedness, rotations never do.

With a motif: down to the minimum

A bent molecule keeps one mirror family



Hermann–Mauguin: **m**

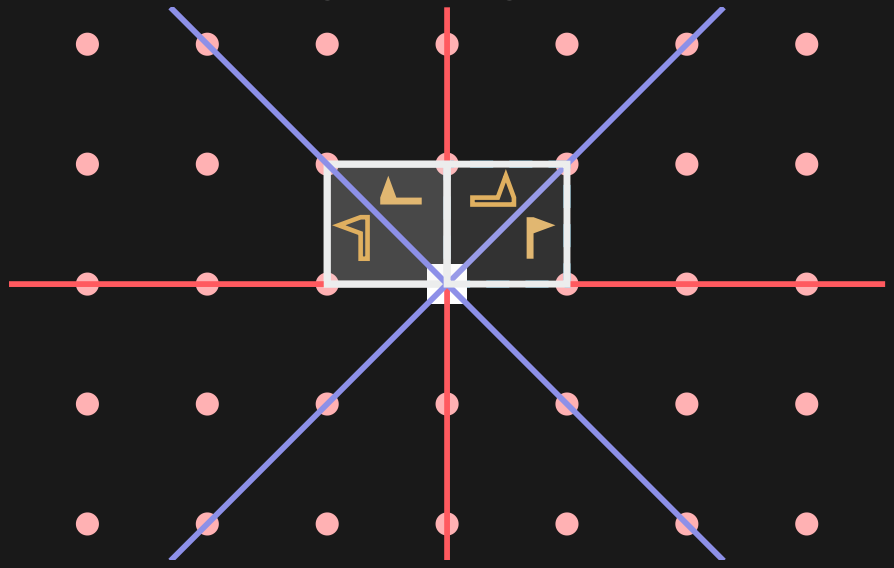
Per *Solid State Materials Chemistry*, §§1.1.1–1.1.2.

SQUARE: 4 ↔ 4mm

a = b, γ = 90°

Lattice alone: the maximum

Fourfold plus edge and diagonal mirror families

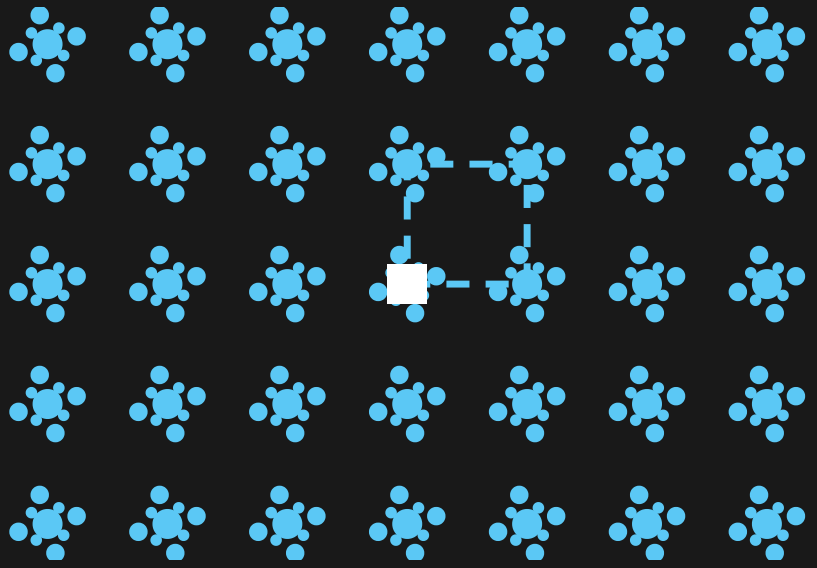


Hermann–Mauguin: **4mm**

The turned flags stay right-handed; each m makes a copy no rotation can.

With a motif: down to the minimum

A pinwheel molecule keeps only the fourfold



Hermann–Mauguin: **4**

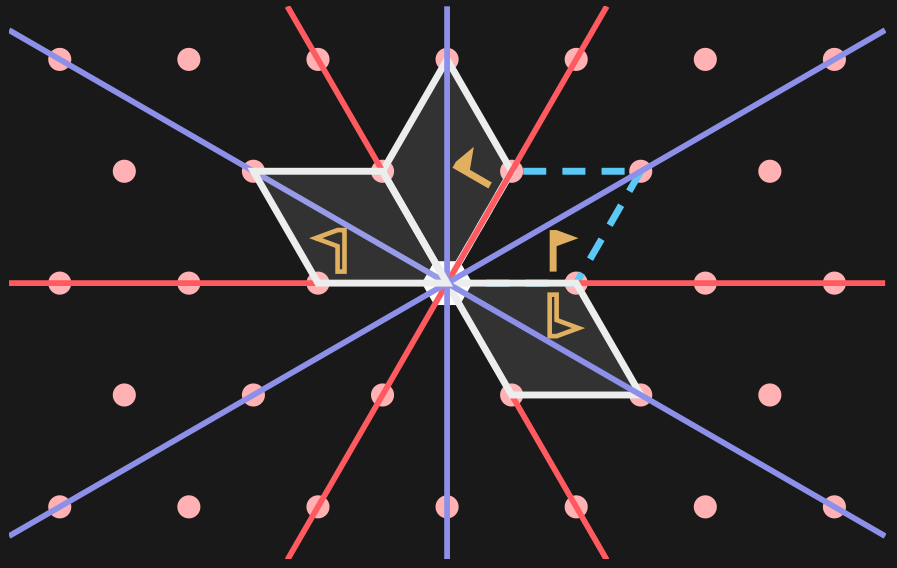
Per *Solid State Materials Chemistry*, §§1.1.1–1.1.2.

HEXAGONAL: 3 ↔ 6mm

a = b, γ = 60°. The 120° cell from lecture 1 describes the same net.

Lattice alone: the maximum

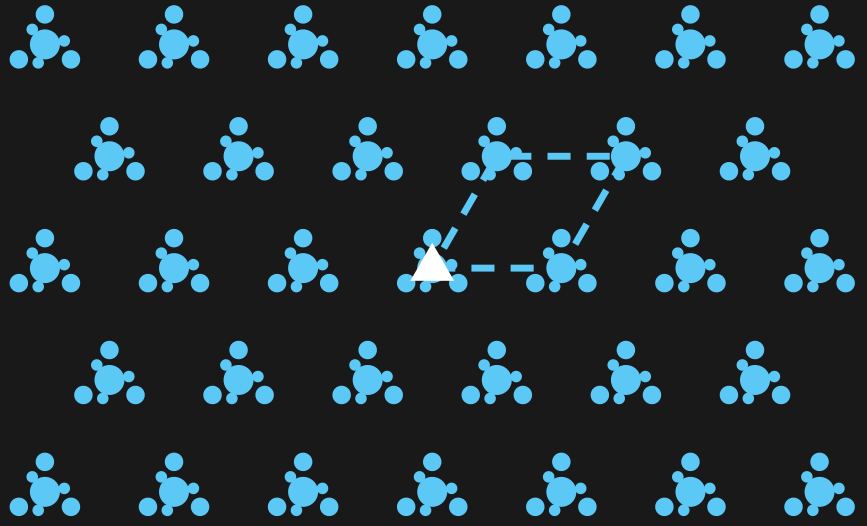
Sixfold plus six mirrors in two alternating families



Hermann–Mauguin: **6mm**

With a motif: down to the minimum

A three-bladed molecule keeps only a threefold



Hermann–Mauguin: **3**

Per *Solid State Materials Chemistry*, §§1.1.1–1.1.2.

SUMMARY: THE FOUR 2D CRYSTAL SYSTEMS

In **4mm** the number is a rotation axis, each **m** a family of mirrors containing it. A system is defined by the point symmetry it can carry: minimum is what membership requires, maximum what the bare lattice has.

2-D crystal systems: minimum ↔ maximum point-group symmetry

Crystal system	Unit-cell metric (a,b,γ)	Minimum symmetry (H–M)	Maximum symmetry (H–M)
Oblique	$a \neq b,\ \gamma \neq 90^\circ, 60^\circ$	**1** (no symmetry)	**2** (one twofold rotation)
Rectangular	$a \neq b,\ \gamma = 90^\circ$	**m** (one mirror)	**2mm** (twofold + two perpendicular mirror families)
Square	$a = b,\ \gamma = 90^\circ$	**4** (four-fold only)	**4mm** (fourfold + mirrors along edges and diagonals)
Hexagonal (triangular net)	$a = b,\ \gamma = 60^\circ$	**3** (three-fold only)	**6mm** (sixfold + six mirrors)

A motif can lower the symmetry of a lattice while the lattice retains its own maximum symmetry.

2-D BRAVAIS LATTICES (5 TOTAL)

Bravais lattice	Lattice type	Metric constraints	Notes / equivalences
Oblique (p)	primitive	$a \neq b,\ \gamma \neq 90^\circ, 60^\circ$	General parallelogram.
Rectangular (p)	primitive	$a \neq b,\ \gamma = 90^\circ$	Edges orthogonal, unequal lengths.
Rectangular (c)	**centered**	$a \neq b,\ \gamma = 90^\circ$	Distinct Bravais type. Conventional rectangle has a lattice point at the center; primitive basis can be taken as $\mathbf{u} = \frac{1}{2}(\mathbf{a} + \mathbf{b}), \mathbf{v} = \frac{1}{2}(-\mathbf{a} + \mathbf{b})$ (a rhombus).
Square (p)	primitive	$a = b,\ \gamma = 90^\circ$	Any "centered square" reduces to a primitive square via a 45° rotation and rescale (not a new Bravais type).
Hexagonal / Triangular (p)	primitive	$a = b,\ \gamma = 60^\circ$	Often called "triangular" in 2-D. Any centering is equivalent to primitive.

CRYSTAL SYSTEMS AND BRAVAIS LATTICES

Aspect	Crystal system	Bravais lattice
What is classified?	Sets of **point groups** (rotations/mirrors about a fixed point)	Sets of **vector lattices** (translations) distinguished up to centering
Includes translations?	**No**	**Yes** (pure translations only; point symmetry comes from the lattice metric)
What it fixes	Cell **metric**/shape constraints (e.g., $a = b$, $\gamma = 90^\circ$ for square)	Metric **and** whether a **distinct centering** exists (e.g., primitive vs centered)
2-D inventory	**4 systems:** oblique, rectangular, square, hexagonal	**5 Bravais types:** p-oblique, p-rectangular, **c-rectangular**, p-square, p-hexagonal
3-D inventory	**7 systems:** triclinic, monoclinic, orthorhombic, tetragonal, trigonal, hexagonal, cubic	**14 Bravais types** (e.g., cubic-P, cubic-I, cubic-F; orthorhombic-P/C/I/F; etc.)
Example mapping	"Square system" (requires $a = b,\ \gamma = 90^\circ$)	Within square: only **p-square** (any 'centered square' reduces to p-square). Rectangular admits **p** and **c**.

THE FIVE POINT SYMMETRY ELEMENTS

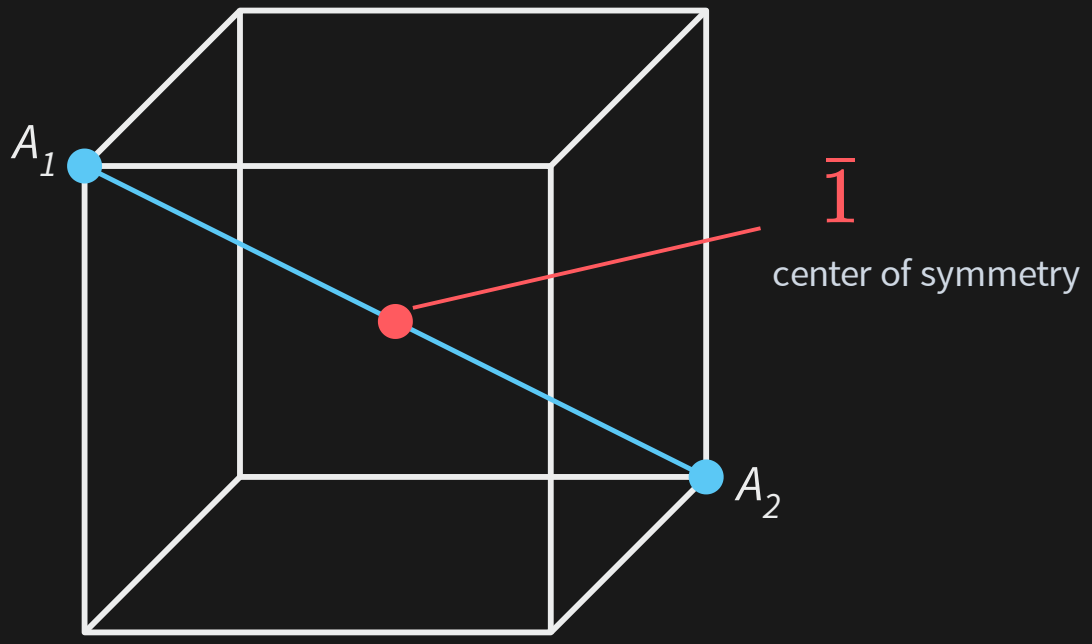
This morning rotations and mirrors sorted every 2D lattice into four systems; every symbol we wrote, like **2mm**, was a list of them. Today's second half sorts the 3D lattices the same way: five elements, two of them new.

Point symmetry element	Symbol	Symmetry operation	vs. 2D
Identity	1	Every point stays in place; every object has it	same
Proper rotation axis	N	Rotate by 360/N° about the axis; only N = 1, 2, 3, 4, 6 fit a lattice	a point becomes an axis
Mirror plane	*m*	Reflect through the plane	a line becomes a plane
Inversion center	$\bar{1}$	Send every point through the center to the opposite side	**new in 3D**
Rotoinversion axis	$\bar{N}$	Rotate by 360/N° about the axis, then invert through a point on it	**new in 3D**

"Point symmetry" means each operation leaves some location unmoved: a rotation moves every point except those on its axis, a mirror leaves its plane, inversion leaves one center. A translation moves every point; the two elements that carry one arrive next meeting.

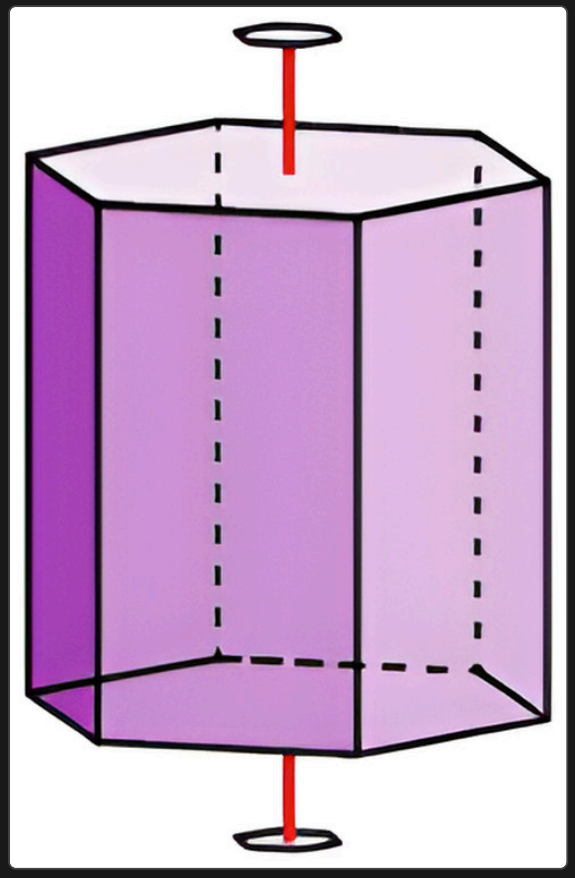
5 elements → groups → 7 crystal systems → 32 point groups

INVERSION CENTER



- Inversion through a point, called the center of symmetry
- The element is written $\bar{1}$, read “one bar”

SIXFOLD AXIS OF ROTATION

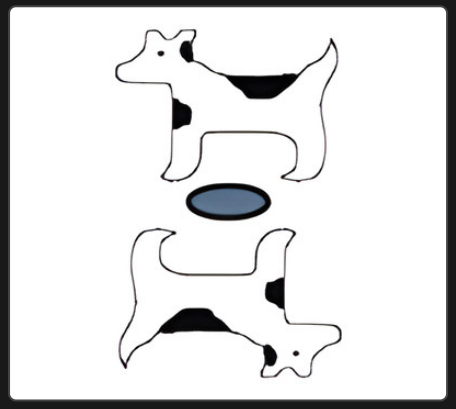


When rotated about its axis, the crystal repeats itself every 60° (six times in a 360° rotation).

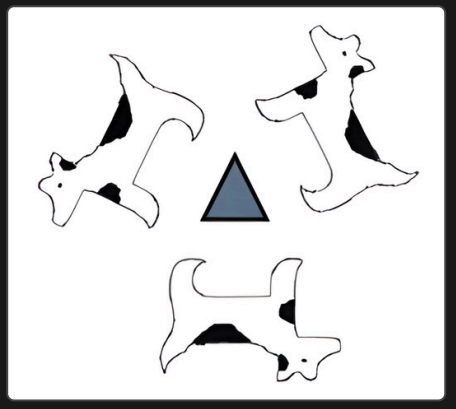
In three dimensions a rotation turns about an axis, a line through the crystal, not a point.

ROTATION AXES: NAMES AND SYMBOLS

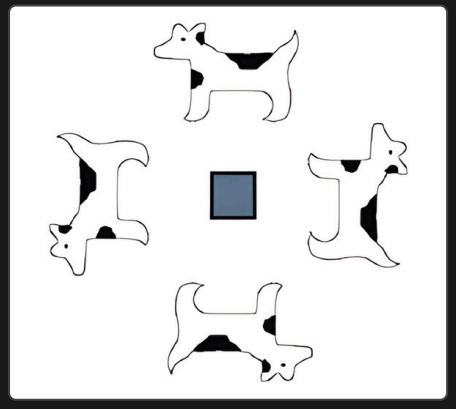
An n fold rotational symmetry operation rotates an object by $360^\circ / n$. Only n = 1, 2, 3, 4, and 6 are permitted in a periodic lattice.



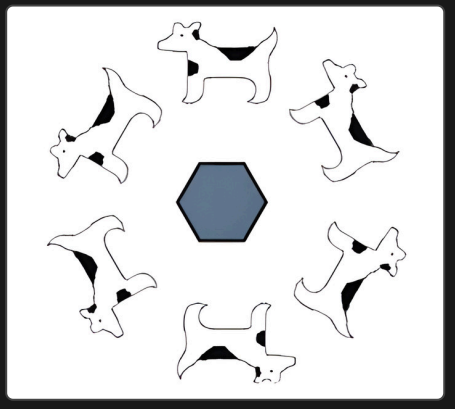
2 fold
‘Diad’



3 fold
‘Triad’



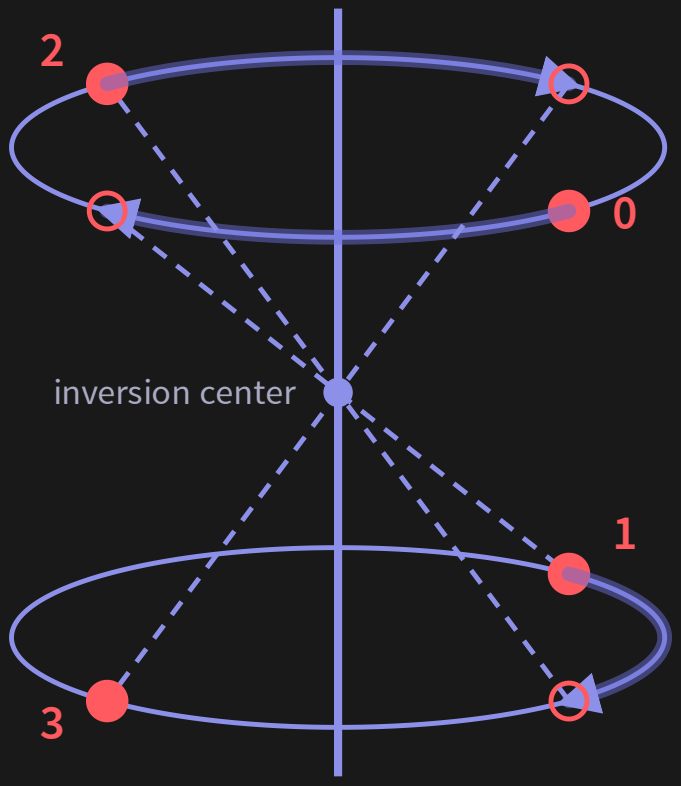
4 fold
‘Tetrad’



6 fold
‘Hexad’

The blue marker at each center is the axis’s graphical symbol: lens, triangle, square, hexagon.

ROTOINVERSION $\bar{N}$



$\bar{4}$: rotate 90°, then invert

$\bar{N}$: rotate by $360^\circ / N$ about the axis, then invert through a point on that axis.

Rotate 90°, invert: $0 \to 1$

Again: $1 \to 2$

Again: $2 \to 3$, then the set closes: four points

The $\bar{N}$ family

$\bar{1}$: inversion center

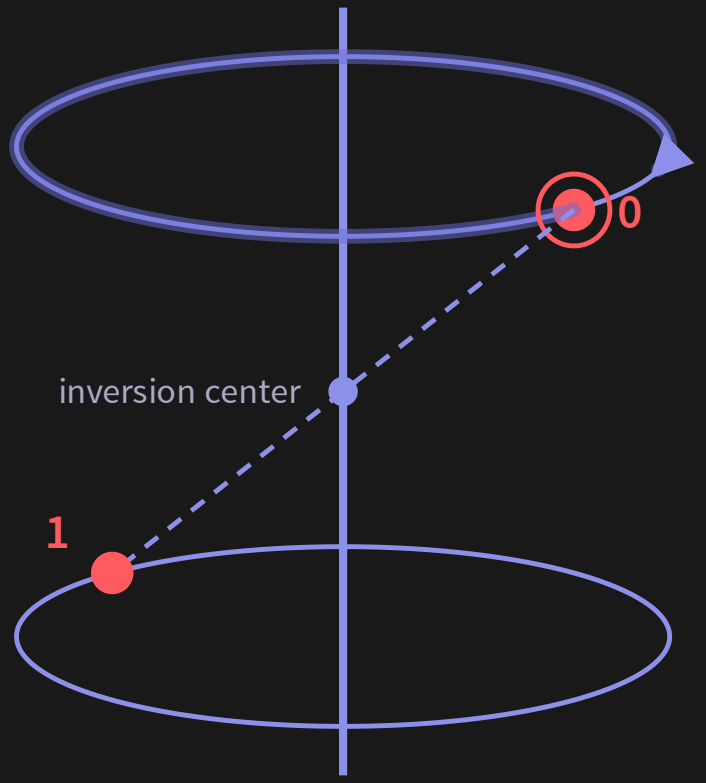
$\bar{2}$: m, a mirror plane

$\bar{3}$: rotate 120°, invert; six points

$\bar{4}$: rotate 90°, invert; four points

$\bar{6}$: rotate 60°, invert; six points

$\bar{1}$: A FULL TURN, THEN INVERT



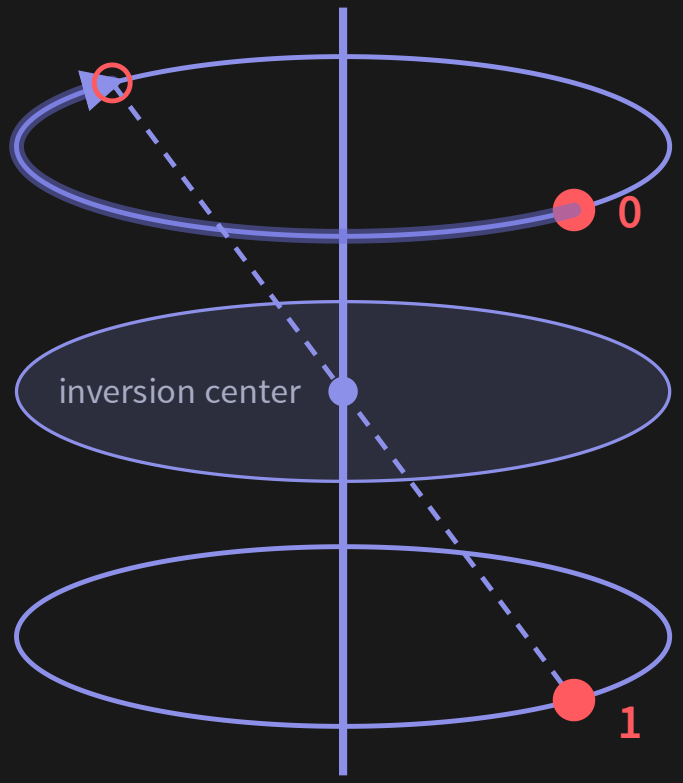
$\bar{1}$: turn by $360°/1$ about the axis, then invert through the center.

Rotate $360°$ (back to start), invert: straight through the center

$\bar{1}$ is the inversion center

$\bar{1} \equiv i$: rotating a full turn changes nothing, so all that is left is the inversion

$\bar{2}$: HALF A TURN, THEN INVERT

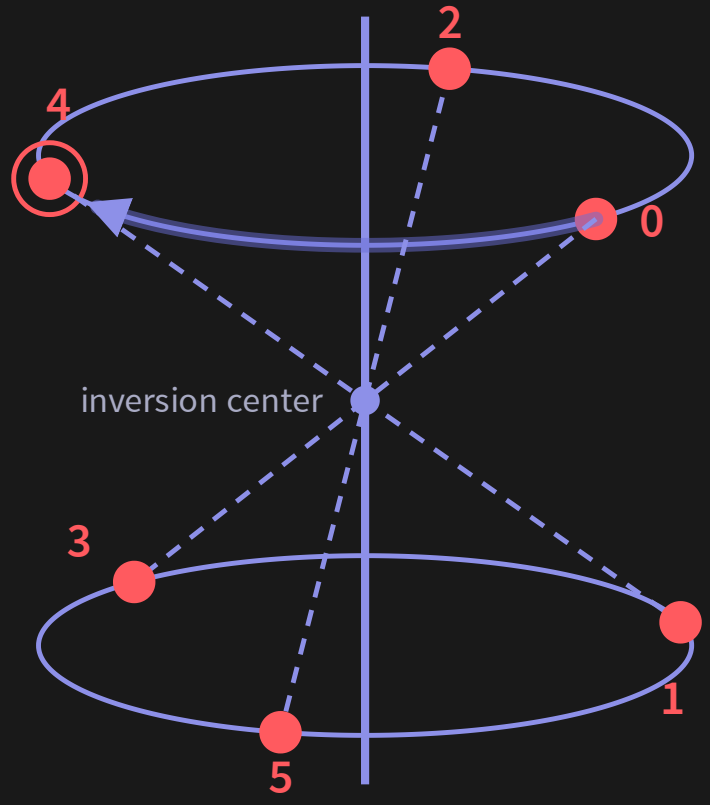


$\bar{2}$: turn by $360^\circ/2$ about the axis, then invert through the center.

Rotate 180°, invert: the point lands straight below

$\bar{2} \equiv m$: a mirror plane perpendicular to the axis

$\bar{3}$: ROTATE 120°, THEN INVERT



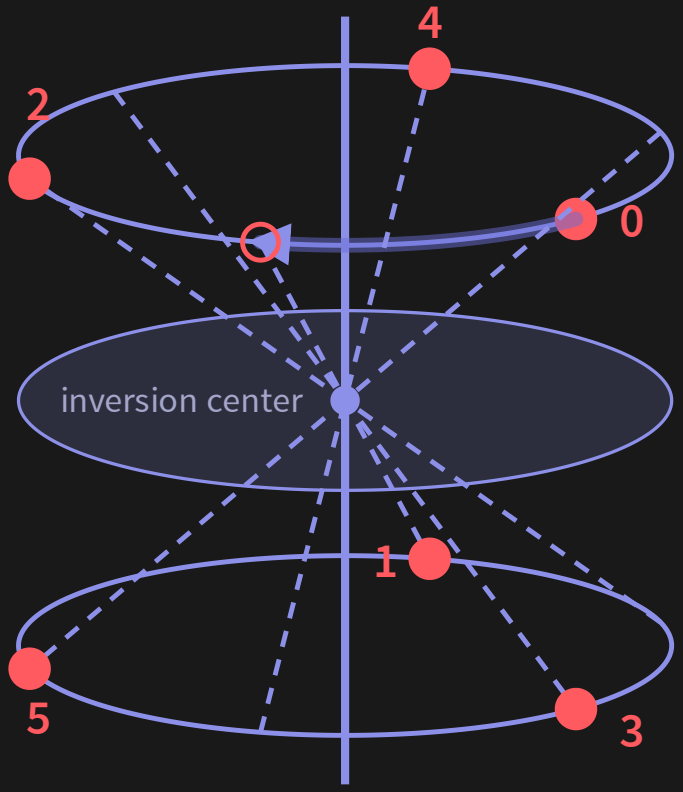
$\bar{3}$: turn by $360^\circ/3$ about the axis, then invert through the center.

Rotate 120°, invert: $0 \to 1$

Six equivalent points before it closes

$\bar{3}$ contains both a threefold and an inversion center

$\bar{6}$: ROTATE 60°, THEN INVERT



$\bar{6}$: turn by $360^\circ/6$ about the axis, then invert through the center.

Rotate 60°, invert: $0 \rightarrow 1$

Two eclipsed triangles, one per circle

$\bar{6} \equiv 3/m$: a threefold axis with a mirror plane perpendicular to that axis

N/m: the slash always makes the mirror perpendicular to the N axis it follows; an m beside the number, as in $4mm$, contains the axis instead

SYMMETRY ELEMENT SYMBOLS

Table 1.28 ***Point symmetry elements***

Symmetry element	Written symbol	Graphical symbol
Rotation axes	1	None
	2	
	3	
	4	
	6	
Inversion axes	$\bar{1}$	None[a]
	$\bar{2}(\equiv m)$	——[b]
	$\bar{3}(\equiv 3+\bar{1})$	
	$\bar{4}$	
	$\bar{6}(=3/m)$	
Mirror plane	m	——

[a] The inversion axis, $\bar{1}$, equivalent to a centre of symmetry, is represented as a small open circle, o, in space groups, but does not have a formal graphical representation in point groups, even though it is present in many point groups.

[b] The inversion axis, $\bar{2}$ does not have a separate graphical symbol other than that of the mirror plane equivalent to it.

From West, Solid State Chemistry and its Applications

The written symbol goes into a point-group or space-group symbol; the graphical symbol marks the element on a crystallographic diagram.

The filled shape counts the order: a lens has two sides, a triangle three, a square four, a hexagon six.

WHY GROUP THE SYMMETRY ELEMENTS?

Lecture 1 sorted cells into seven crystal systems by shape. Shape is the consequence: each system is defined by the symmetry its lattice must carry.

One element at a time is not enough. A crystal's operations act together, and the set must close on itself: a **group**.

7 crystal systems → 32 point groups → × 14 Bravais lattices + screws and glides → 230 space groups → any crystal structure

A published structure is one space-group symbol plus a short list of atoms; symmetry generates the rest.

5 elements → **groups** → 7 crystal systems → 32 point groups

WHY POINT GROUPS? FACES, NOT ATOMS

The 32 classes were found before atoms could be seen, from two laws of faces, measured with a goniometer.

Steno 1669	the angles between a mineral's faces are constant
Haüy 1784	faces sit at orientations given by small whole numbers



Fluorite — cubic



Pyrite — cubic, striations break the fourfold



Quartz — trigonal, left- and right-handed forms



Gypsum — monoclinic

Faces come in symmetry-related sets: the class is readable from the shape.

Symmetries never come one at a time: two operations in a row are some third operation, so a crystal's operations form a closed set.

Hessel (1830) counted the closed sets the face laws allow: exactly **32**. Unnoticed; rederived by Gadolin (1867), 80 years before X-rays.

5 elements → **groups** → 7 crystal systems → 32 point groups

POINT GROUPS

A point group is a set of symmetry operations, acting on an isolated object, that fulfils the four requirements of a group.

A group must have

closure
two combined give one of the set

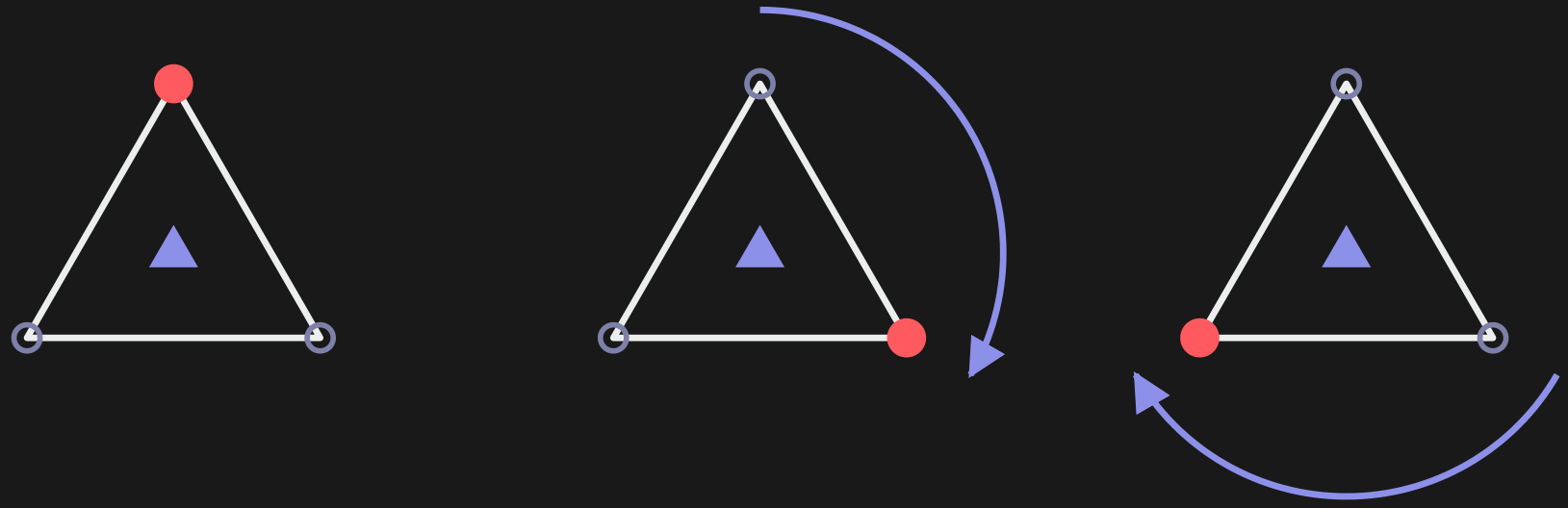
the associative law
any pair in a chain may go first

an identity
the do-nothing operation, written 1

an inverse
every operation has one that undoes it

CLOSURE

This triangle has exactly three operations: do nothing (1), rotate 120°, and rotate 240°.



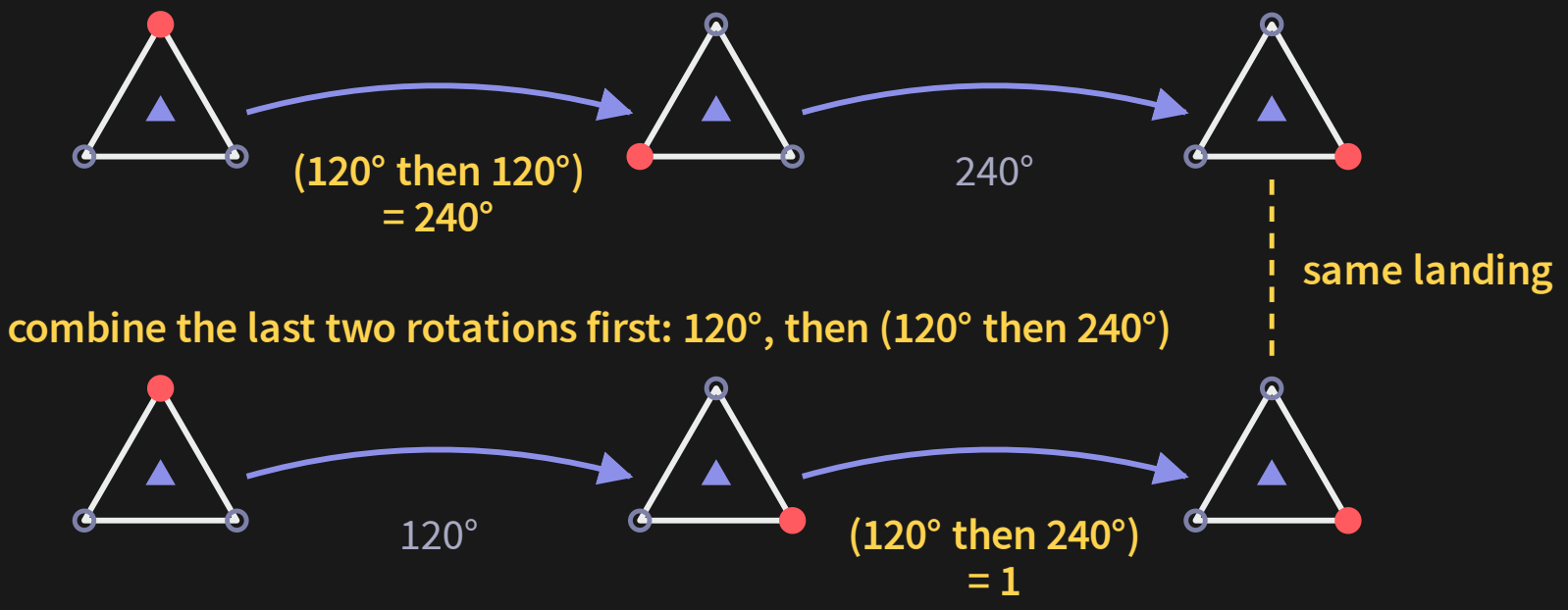
start **rotate 120°** **and 120° again**

120° then 120° is the same net move as a single 240° rotation, and 240° is one of the three operations above. Combining two of them gave another one: closure.

Each operation lands the triangle back on its own outline. The red dot is not part of the triangle; it tells the three identical-looking positions apart.

THE ASSOCIATIVE LAW

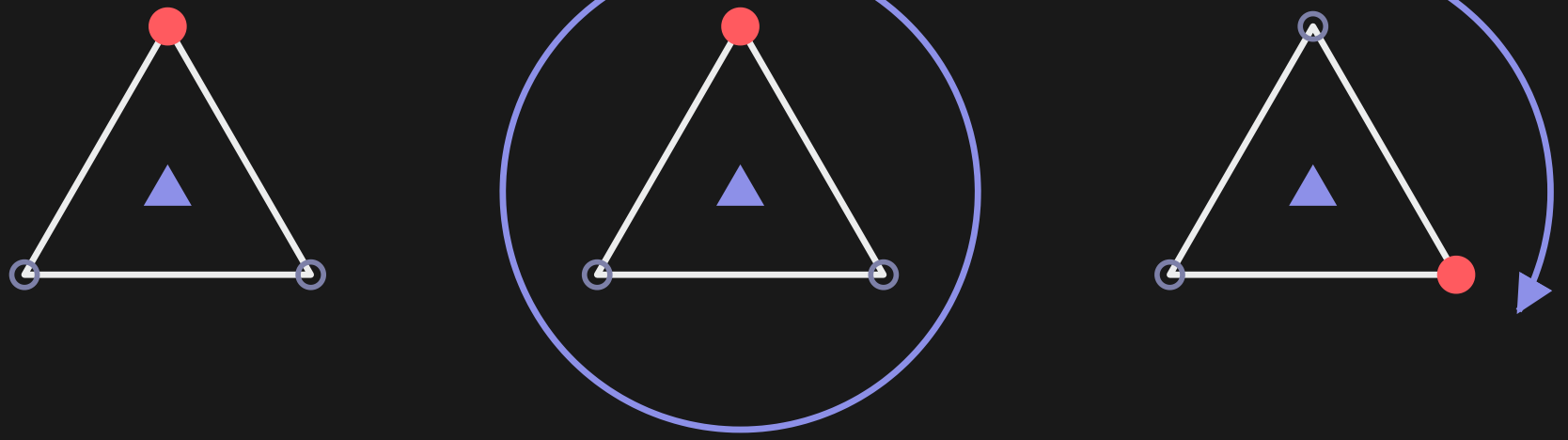
one chain, fixed order: 120°, then 120°, then 240°



Both groupings land the marked corner in the same place.

The three rotations happen in the same order both times. The only choice is which two neighbors to merge into a single move first, and either choice gives the same result.

IDENTITY



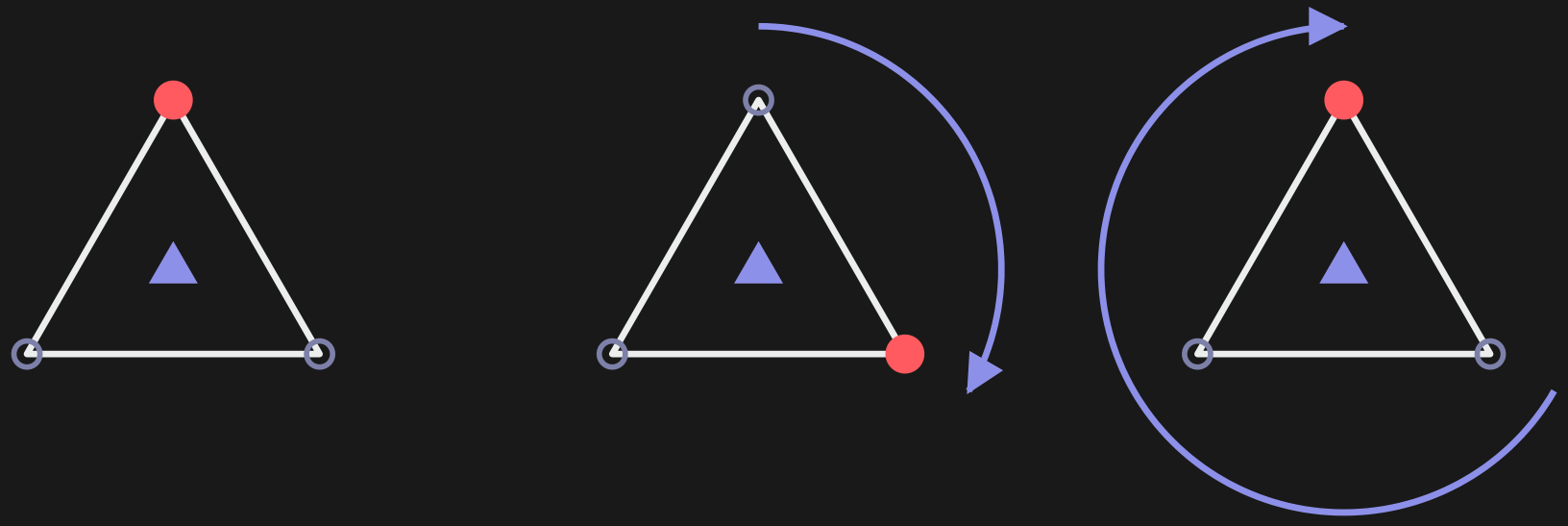
do nothing: 1

a full 360° turn

1, then 120°

Combining 1 with any operation leaves that operation unchanged.

INVERSE



start rotate 120° then rotate 240°

120° and 240° undo each other: combined, they give the identity, so each is the inverse of the other.

An inverse is a partner operation, and every operation in a group has one. The inversion center $\bar{1}$ is a different thing: one particular symmetry operation, which happens to be its own inverse.

THE TRIANGLE'S POINT GROUP

The set: 1, rotate 120°, rotate 240°.

✓ closure: two rotations combined always gave a rotation of the set

✓ associative law: whichever neighboring pair we merged first, the corner landed in the same place

✓ identity: the do-nothing operation, 1, is in the set

✓ inverse: 120° and 240° undo each other, and 1 undoes itself

All four requirements hold, so these three rotations are a point group. Its symbol is **3**: a single threefold axis.

The triangle also has three mirror planes through the axis; adding their operations gives a larger point group, 3m.

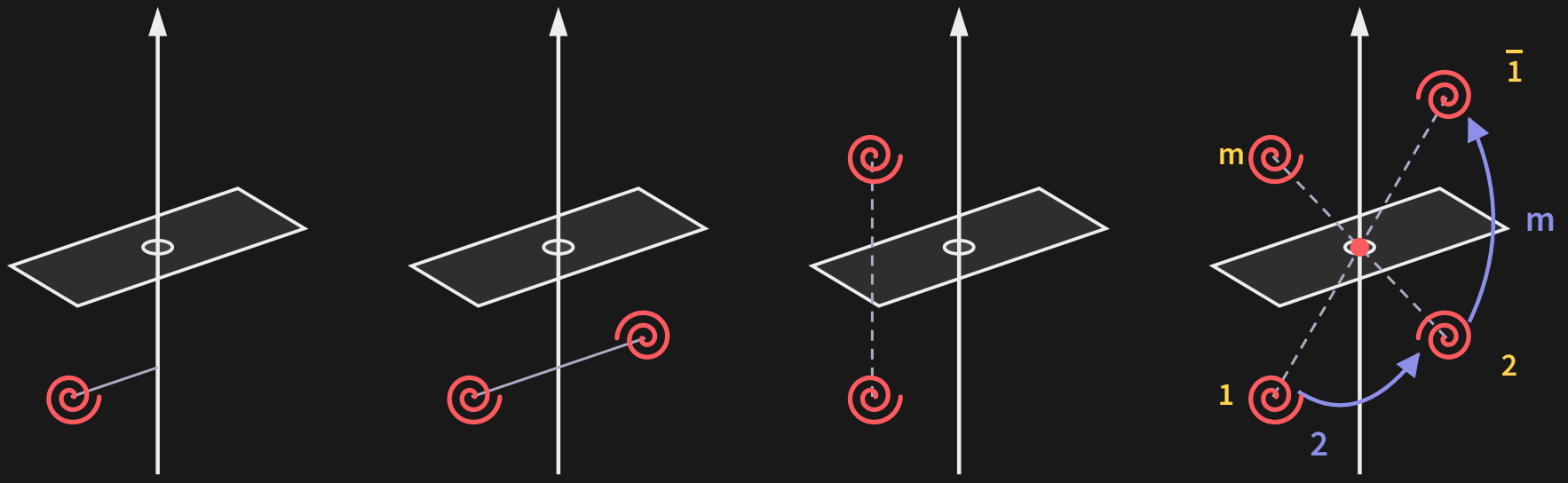
QUESTION

Let's take a twofold rotation axis. And let's have a mirror plane perpendicular to it.

Is that a valid point group, just those two symmetry operations?

If not, what other symmetry operations do we need to add to those two to get a valid point group?

POINT GROUP $2/m$



Four operations, four copies of the motif: 1 left it in place, 2 rotated it, m reflected it, and the new diagonal jump is $\bar{1}$, inversion through the center. Closure adds that inversion — the group is $\{1,\ 2,\ m,\ \bar{1}\}$, written **2/m**: a twofold axis with a mirror plane perpendicular to it.

THE SEVEN CRYSTAL SYSTEMS

A crystal system is defined by the point symmetry it can carry; the cell shape follows.

This morning · two dimensions · four systems

oblique	rectangular	square	hexagonal
up to 2	up to 2mm	up to 4mm	up to 6mm

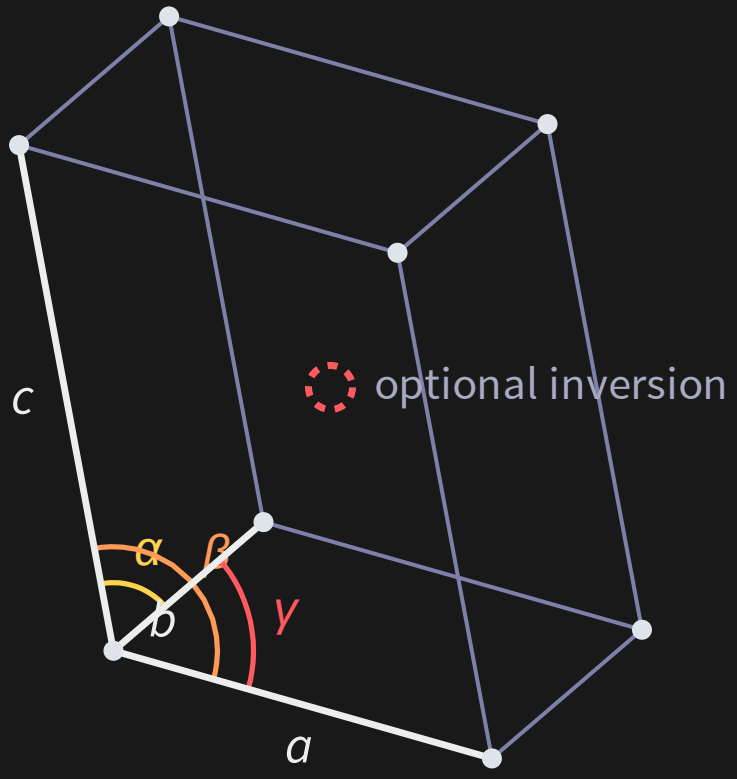
Now · three dimensions · seven systems, least required symmetry to most

triclinic	monoclinic	orthorhombic	tetragonal	trigonal	hexagonal	cubic
nothing required	one twofold	three twofolds	a fourfold	a threefold	a sixfold	four threefolds

Each slide that follows names one system's requirement and collects its point groups; the running count lands on 32.

5 elements → groups → **7 crystal systems** → 32 point groups

TRICLINIC: NOTHING REQUIRED



Triclinic

edges and angles all free

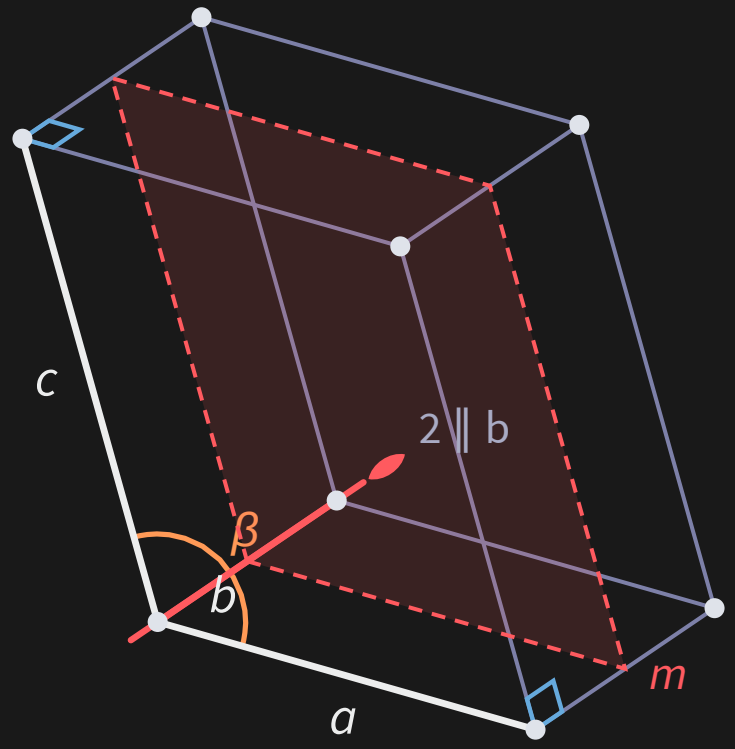
identity only; at most an inversion center: $\bar{1}$

Drawn proportions are representative; the labels state only the required metric constraints.

Per *Solid State Materials Chemistry*, §1.1.3.

Point groups: $1, \bar{1}$ · 2 of 32

MONOCLINIC: ONE TWOFOLD AXIS



Monoclinic

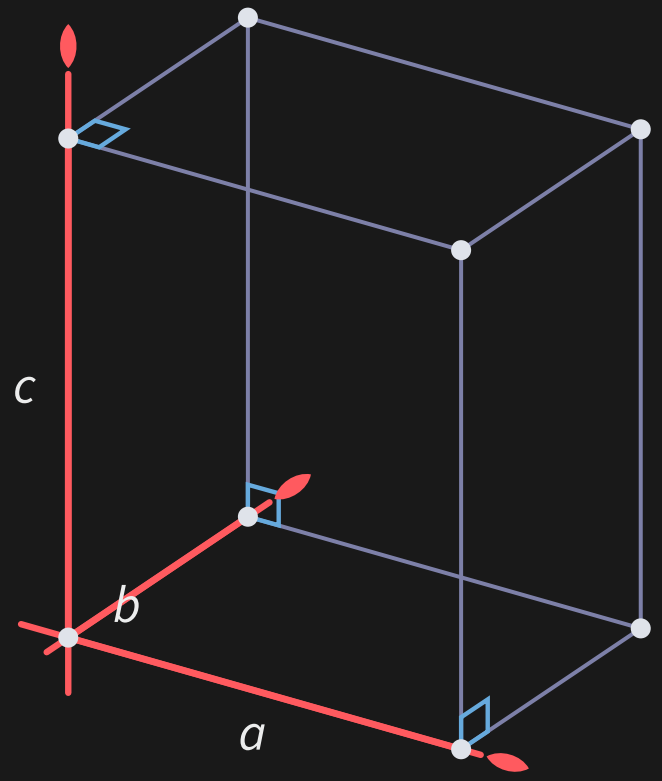
α = γ = 90°; β free

one twofold axis along b, or a mirror plane ⊥ b

Per *Solid State Materials Chemistry*, §1.1.3.

Point groups: $2, m, 2/m$ · 5 of 32

ORTHORHOMBIC: THREE TWOFOLD AXES



Orthorhombic

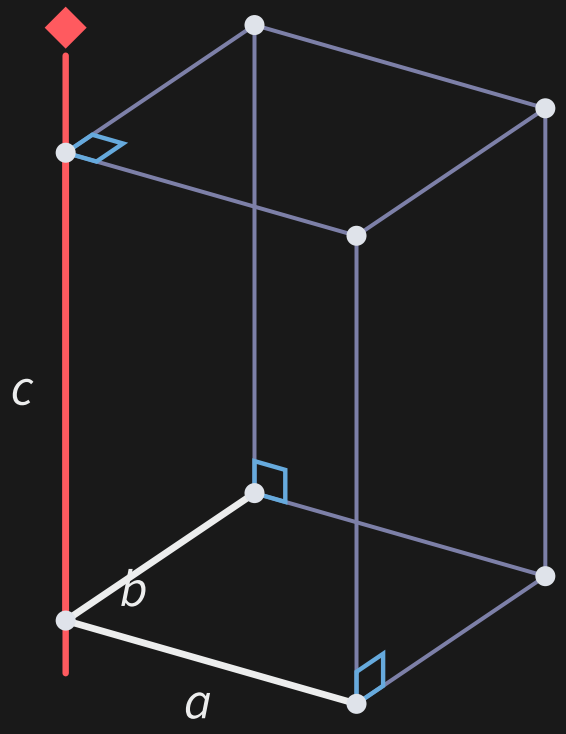
α = β = γ = 90°

three twofold axes,
one along each edge

Per *Solid State Materials Chemistry*, §1.1.3.

Point groups: 222, $mm2$, mmm · 8 of 32

TETRAGONAL: A FOURFOLD ALONG C



Tetragonal

a = b

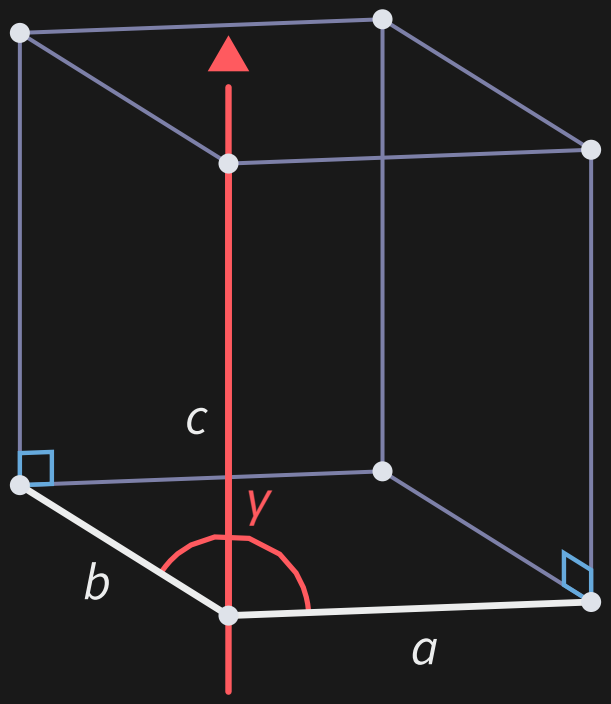
α = β = γ = 90°

one fourfold axis along c: 4, or the rotoinversion $\bar{4}$

Per *Solid State Materials Chemistry*, §1.1.3.

Point groups: $4, \bar{4}, 4/m, 422, 4mm, \bar{4}2m, 4/mmm$ · 15 of 32

TRIGONAL: A THREEFOLD ALONG C



Trigonal

hexagonal setting

a = b; α = β = 90°

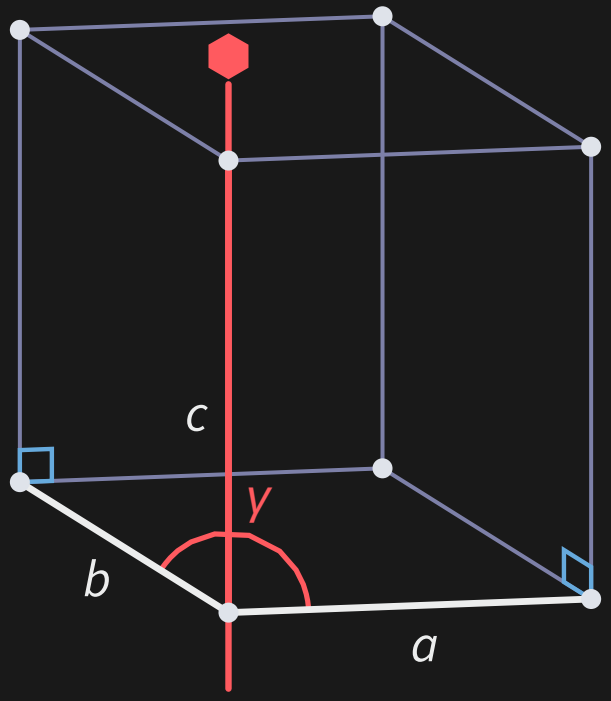
γ = 120°

one threefold axis along c: 3 or $\bar{3}$

Per *Solid State Materials Chemistry*, §1.1.3.

Point groups: $3, \bar{3}, 32, 3m, \bar{3}m$ · 20 of 32

HEXAGONAL: A SIXFOLD ALONG C



Hexagonal

a = b; α = β = 90°

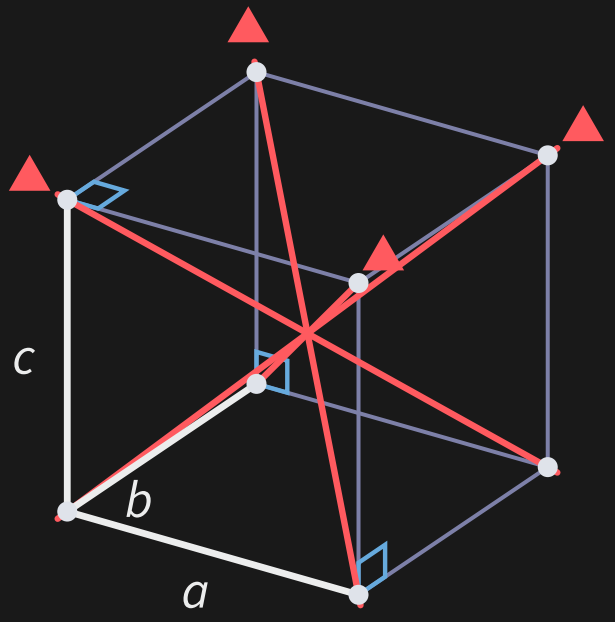
γ = 120°

one sixfold axis along c: 6 or $\bar{6}$

Per *Solid State Materials Chemistry*, §1.1.3.

Point groups: $6, \bar{6}, 6/m, 622, 6mm, \bar{6}m2, 6/mmm$ · 27 of 32

CUBIC: FOUR THREEFOLDS



Cubic

a = b = c

α = β = γ = 90°

four threefold axes, one along each body diagonal

Per *Solid State Materials Chemistry*, §1.1.3.

Point groups: $23, m\bar{3}, 432, \bar{4}3m, m\bar{3}m$ · 32 of 32

WHY EXACTLY 32?

Lecture 1: only the rotations $N = 1, 2, 3, 4, 6$ can repeat in a crystal. Each row puts something new on those axes, mirrors or twofold axes, and counts the groups; a symbol names the starting elements, so $2/m$ is the four operations $\{1,\ 2,\ m,\ \bar{1}\}$.

What sits on the axis	Point groups	Count
Nothing: the axis by itself is already a group	$1, 2, 3, 4, 6, \bar{1}, m = \bar{2}, \bar{3}, \bar{4}, \bar{6}$	10
A mirror plane perpendicular to the axis: N/m	$2/m, 4/m, 6/m$ ~~$3/m$~~ = $\bar{6}$, already counted on #/17	3
Mirror planes containing the axis: Nmm	$mm2, 3m, 4mm, 6mm$	4
Twofold axes perpendicular to the axis: $N22$	$222, 32, 422, 622$	4
Perpendicular mirror and twofold axes at once: $N/m\,2/m\,2/m$	mmm (short for $2/m\,2/m\,2/m$), $\bar{6}m2, 4/mmm, 6/mmm$	4
A rotoinversion axis carrying twofolds and bisecting mirrors	$\bar{3}m, \bar{4}2m$ $\bar{1}, \bar{2}, \bar{6}$ this way only re-make groups already counted	2
Four threefolds along the body diagonals	$23, m\bar{3}, 432, \bar{4}3m, m\bar{3}m$	5

Symbol order is direction order: mm2 runs x, y, z with its twofold along z; a unique high-order axis comes first, as in 4mm and 6mm.

$$10 + 3 + 4 + 4 + 4 + 2 + 5 = 32$$

Groups per *Solid State Materials Chemistry*, Table 1.1, §1.1.3; the derivation follows Gadolin (1867).

32 CRYSTALLOGRAPHIC POINT GROUPS

5 elements → groups → 7 crystal systems → **32 point groups**

Crystal System	Unit Cell	Required symmetry	Point groups	In the symbol
Cubic	Cubic	3-fold axes along body diagonal	$23, m\bar{3}, \bar{4}3m, 432, m\bar{3}m$	3 or $\bar{3}$ in the **second** position
Tetragonal	Tetragonal	4-fold axis	$\boxed{4}, \bar{4}, 4/m, 422, \boxed{4mm}, \bar{4}2m, 4/mmm$	4 or $\bar{4}$ first
Hexagonal	Hexagonal	6-fold axis	$\boxed{6}, \bar{6}, 6/m, 622, \boxed{6mm}, \bar{6}m2, 6/mmm$	6 or $\bar{6}$ first
Trigonal	Hexagonal or Rhombohedral	3-fold axis	$\boxed{3}, \bar{3}, 32, 3m, \bar{3}m$	3 or $\bar{3}$ first
Orthorhombic	Orthorhombic	Three mutually perpendicular 2-fold axes or mirror planes	$222, \boxed{mm2}, mmm$	only 2 and m, three positions
Monoclinic	Monoclinic	2-fold axis or mirror plane	$\boxed{2}, \boxed{m}, \boxed{2/m}$	a single 2, m, or $2/m$
Triclinic	Triclinic	none	$\boxed{1}, \bar{1}$	1 or $\bar{1}$ alone

We built every tinted group earlier today; the rectangular lattice's $2mm$ appears here in its three-dimensional spelling $mm2$.

32 leads with the 3: trigonal, a threefold with perpendicular twofolds. 23 has the 3 second: cubic, threefolds on the body diagonals.

WHO CARES ABOUT POINT GROUPS?

The symbol is shorthand for the full set of symmetries: say $2/m$ or $m\bar{3}m$ and an expert knows every operation present.

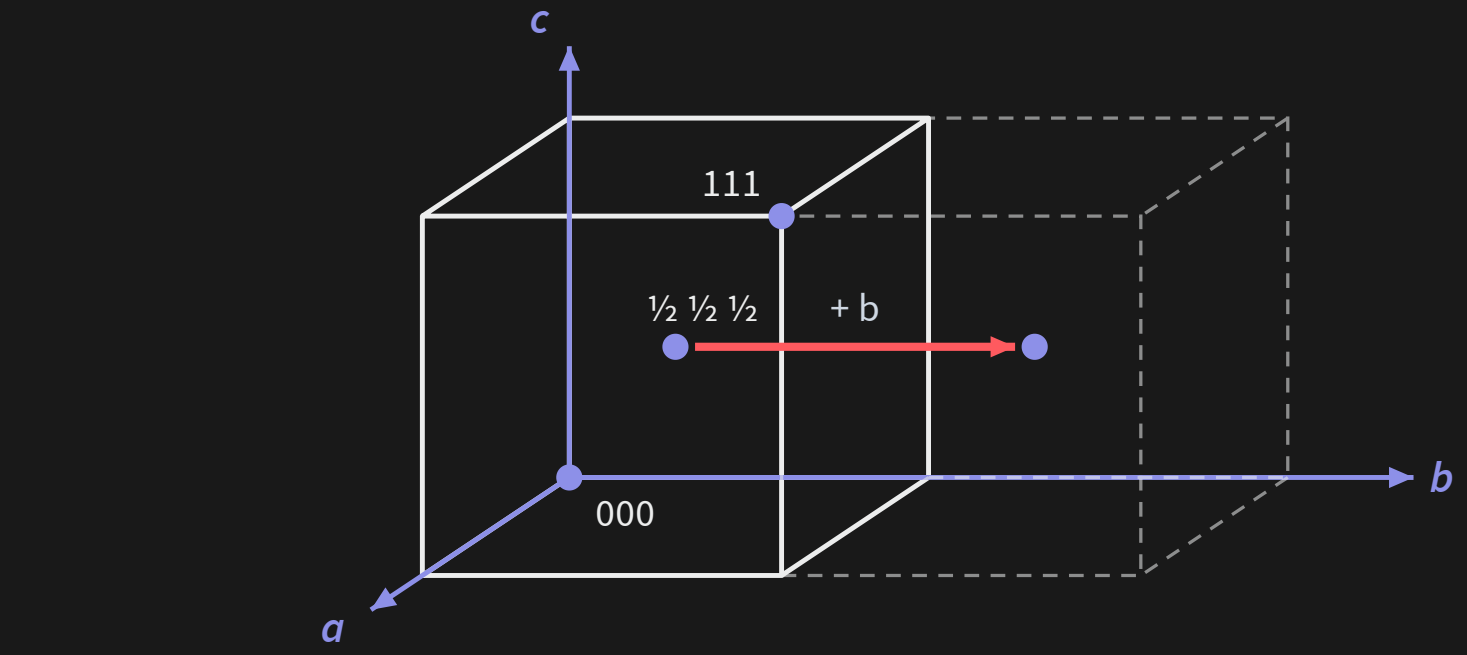
Optical, electrical, and mechanical properties depend directly on that symmetry

Piezoelectricity	only in non-centrosymmetric point groups
Ferroelectricity	only in certain polar point groups
Birefringence	set by the rotation axes that constrain the refractive indices

The point group rules whole classes of properties in or out before any measurement.

POINT COORDINATES

A point inside the cell is named by its coordinates along a, b, and c, as fractions of the cell edges: the origin corner is 000 and the opposite corner is 111.



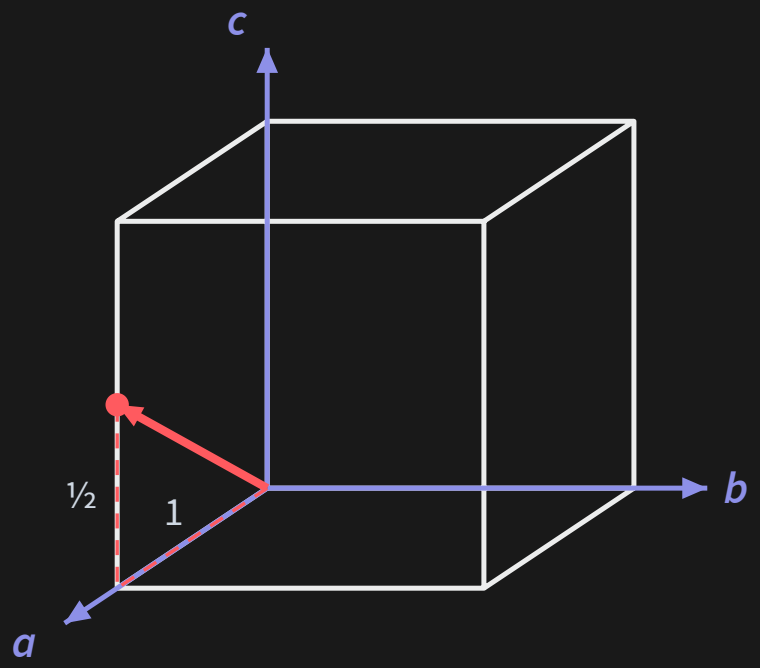
Point coordinates for the unit cell center are $a/2,\ b/2,\ c/2$ — written $\frac{1}{2}\ \frac{1}{2}\ \frac{1}{2}$.

Translation by an integer multiple of the lattice constants reaches the identical position in another unit cell.

CRYSTALLOGRAPHIC DIRECTIONS

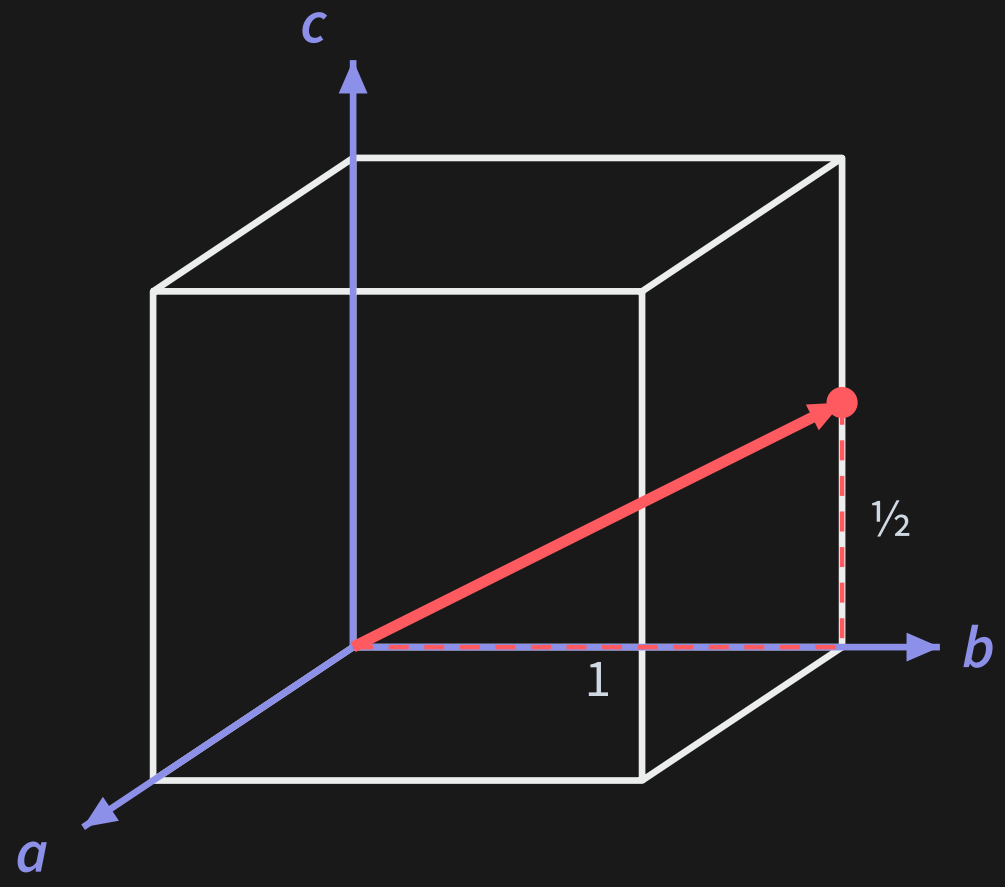
1. Reposition the vector, if necessary, so that it passes through the origin.
2. Read off its projections in terms of the unit cell dimensions $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$.
3. Adjust to the smallest integer values.
4. Enclose in square brackets, no commas: $[uvw]$.

An overbar represents a negative index: $[\bar{1}11]$.



	a	*b*	*c*
Projections	1	0	½
×2	2	0	1
Direction		[201]	

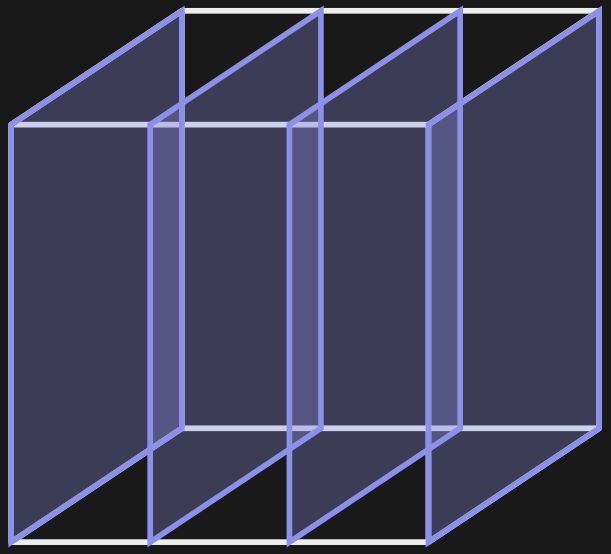
WHAT DIRECTION IS SHOWN BELOW?

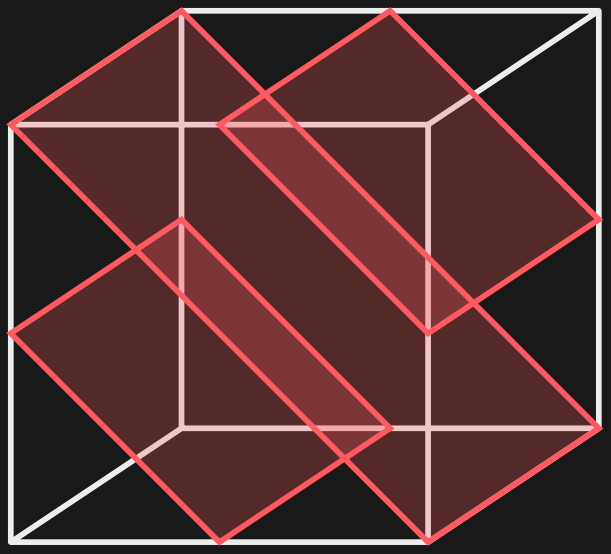


Projections $0,\ 1,\ \frac{1}{2}\ \times 2 \rightarrow [021]$

SETS OF EQUIDISTANT PARALLEL PLANES

Slice through the lattice and the same slice repeats, parallel to itself, all the way through the crystal.





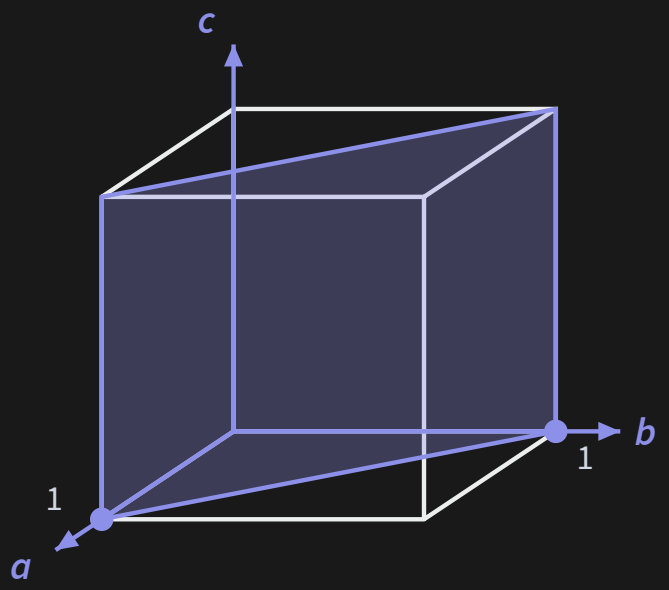
Every plane of a family shares one orientation and one spacing, so naming one of them names all of them.

Tilt the plane and you have a different family, with a spacing of its own.

MILLER INDICES

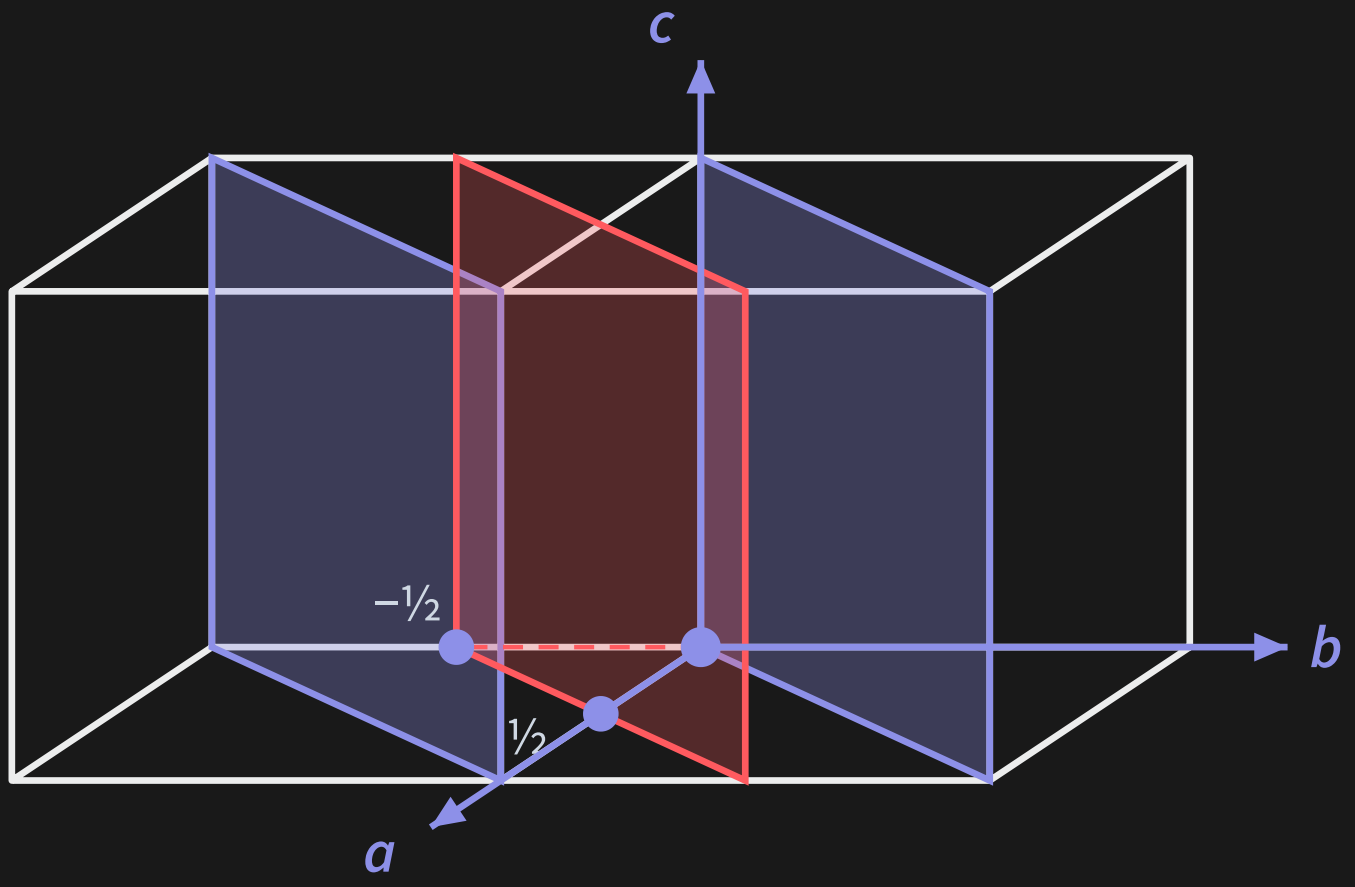
Sets of equidistant parallel planes in a crystalline lattice are represented as (hkl). These values are often called Miller Indices.

1. Read off the intercepts of the plane with the axes in terms of $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$, taking the plane next to the plane that goes through the origin.
2. Take reciprocals of the intercepts: h = to 1/($\mathbf{a}$ intercept), k = to 1/($\mathbf{b}$ intercept), l = to 1/($\mathbf{c}$ intercept).
3. Reduce to the smallest integer values.
4. Enclose in parentheses, no commas: (hkl).



	a	*b*	*c*
1. Intercepts	1	1	∞
2. Reciprocals	1/1	1/1	1/∞
3. Reduction	1	1	0
4. Miller indices		(110)	

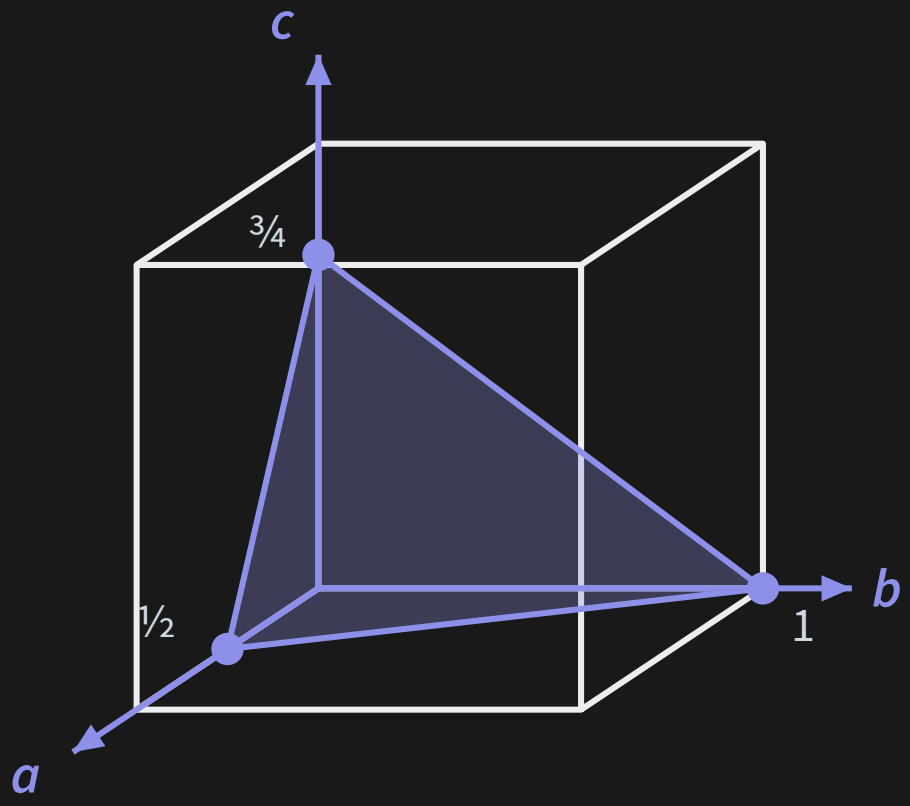
WHAT ARE THE MILLER INDICES?



Intercepts $\frac{1}{2},\ -\frac{1}{2},\ \infty$ Reciprocals $2,\ -2,\ 0 \rightarrow (2\bar{2}0)$

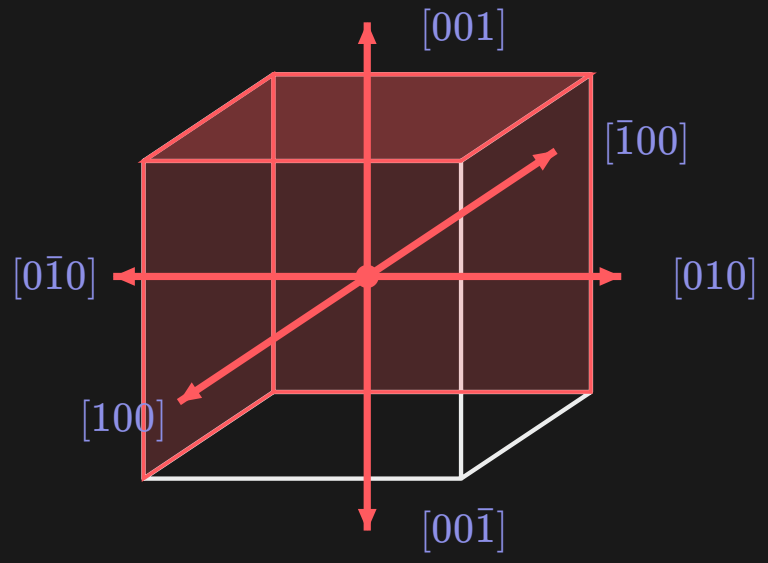
A PLANE THAT NEEDS REDUCING

This plane cuts all three axes, and not one of the intercepts is a whole cell.



	a	*b*	*c*
1. Intercepts	1/2	1	3/4
2. Reciprocals	2	1	4/3
3. Reduction ×3	6	3	4
4. Miller indices		(634)	

SYMMETRICALLY EQUIVALENT DIRECTIONS AND LATTICE PLANES



In certain lattices symmetry makes different directions equivalent.

$\langle 100 \rangle$	the equivalent directions of a cubic crystal: $[100], [010], [001], [\bar{1}00], [0\bar{1}0], [00\bar{1}]$
$\{100\}$	the same for planes: $(100), (010), (001), (\bar{1}00), (0\bar{1}0), (00\bar{1})$

$[uvw]$ one direction · $\langle uvw \rangle$ its equivalent family · (hkl) one family of parallel planes · $\{hkl\}$ the plane families symmetry makes equivalent

PRACTICE

Work the Meeting 2 question set on the course website.

Practice questions →