CHEM 548: MATERIALS CHEMISTRY

LECTURE 1: TRANSLATIONAL SYMMETRY IN 2D

Mon 08/24/2026

Course overview and logistics; translational symmetry in 2D and 3D, lattices, unit cells, and the 14 Bravais lattices

Reading: Woodward, Karen, Evans, and Vogt, *Solid State Materials Chemistry* Ch. 1 §1.1, pp. 1–12, and Problem 1.2 (§1.6)

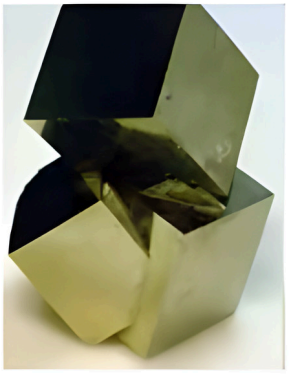
THE COURSE WEBSITE

chem548.wileylab.org

- Course roadmap — every topic we cover, as one map
- Syllabus
- Class schedule
- Lectures
- Practice
- Stats

CRYSTALLINE VS. AMORPHOUS SOLIDS

Crystalline vs. Amorphous Solids



Iron pyrite
(FeS_2)
a crystalline solid



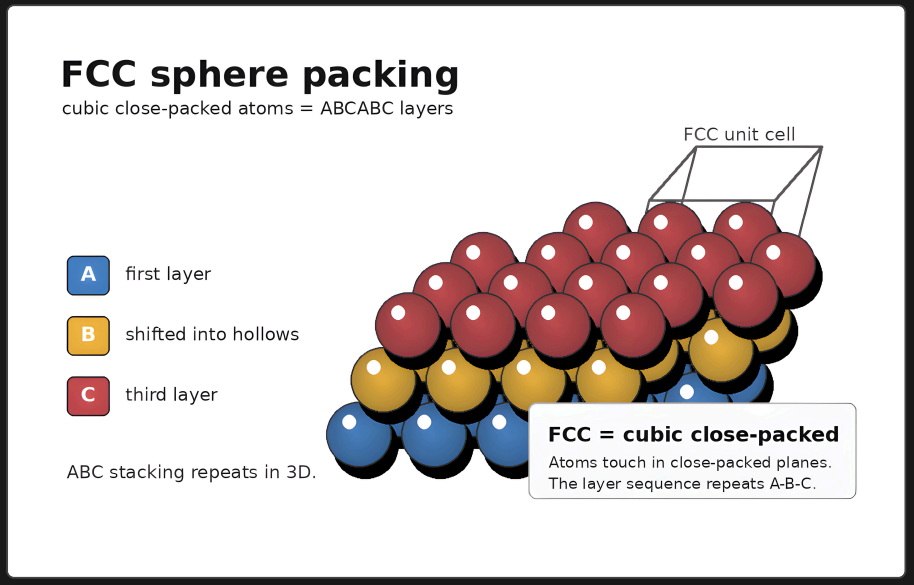
Obsidian
(typically $KAlSi_3O_8$)
an amorphous solid

Crystalline solids possess translational symmetry, amorphous solids do not
(they are like frozen liquids)

This course is about crystalline solids — one repeating unit tells you the whole structure.

A COMPREHENSIVE MATHEMATICAL FRAMEWORK FOR DESCRIBING CRYSTALS

- One intuitive description: **atoms packed as spheres**.
- FCC packing shows close-packed planes with **ABC** stacking.
- But sphere pictures do **not** scale to all possible crystals.
- **We need the *minimum* mathematical framework for *all* crystals.**



TRANSLATIONAL SYMMETRY IN 2D

Why start here

- Every crystal in this course is a **lattice + motif** — one repeating pattern
- Master the pattern language and properties follow: which defects can form, how phases arrange, how electrons move
- 2D is where every idea is visible at a glance

Learning Objectives

- Differentiate crystalline vs amorphous solids
- Define lattice, vectors, unit cell
- Identify shapes that tile space
- Recognize allowed rotation axes (2, 3, 4, 6-fold)
- See how lattice + motif → crystal (e.g. graphene)

Relationship to Text

Solid State Materials Chemistry, Ch. 1: Structures of Crystalline Materials — assigned §1.1 (pp. 1–12)

- 1.1.1 Translational Symmetry (pp. 2–3)
- 1.1.2 Rotational Symmetry (pp. 3–5)
- 1.1.3 Crystallographic Point Groups and Crystal Systems (p. 5)
- 1.1.4 Bravais Lattices (pp. 5–8)
- 1.1.5 Introduction to Space Groups (pp. 8–9)
- 1.1.6 Symmetry Elements That Combine Rotation and Translation (pp. 9–11)
- 1.1.7 Space-Group Symbols (pp. 11–12)

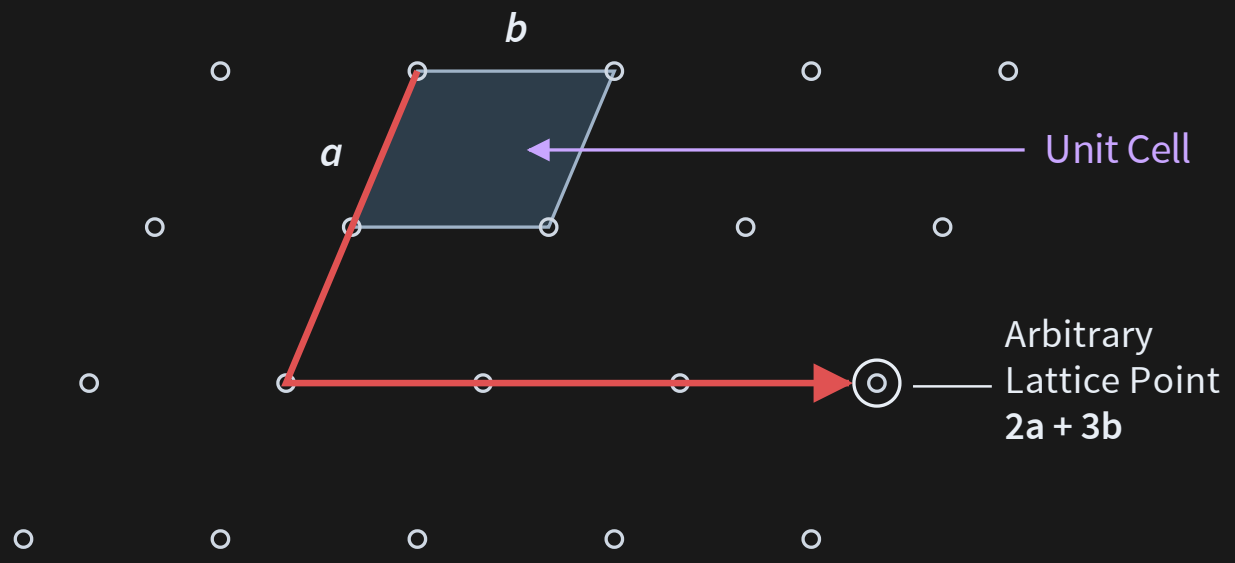
WE START FROM THE BEGINNING

- **Warning:** section 1.1 of the book is very **DENSE** and difficult to **UNDERSTAND**.
- It is an attempt to condense several chapters of crystallography into a few pages.
- We will **EXPAND** upon this material in the next few lectures.

Other helpful online resources:

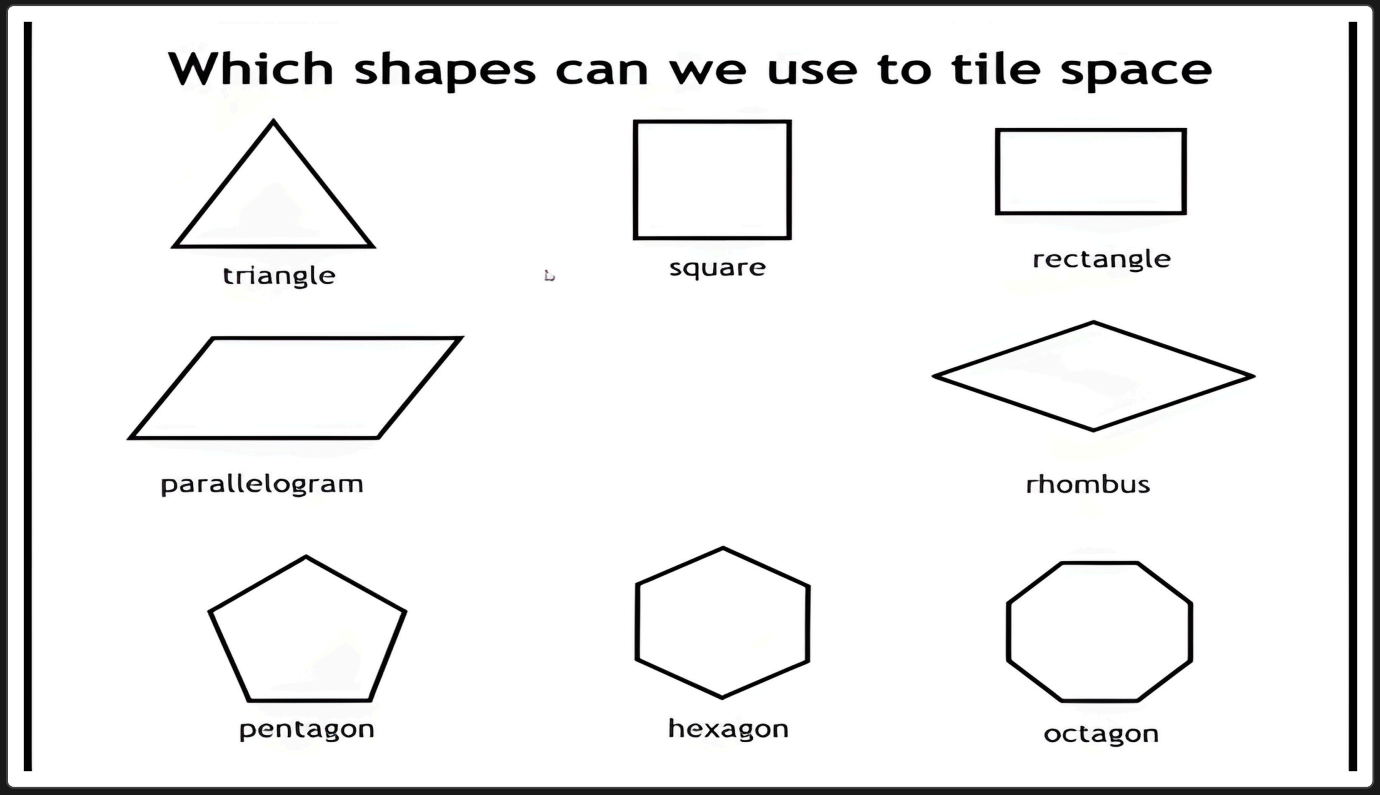
- West, *Solid State Chemistry and its Applications*
- Hammond, *The Basics of Crystallography and Diffraction*

TRANSLATIONAL SYMMETRY (IN 2D)

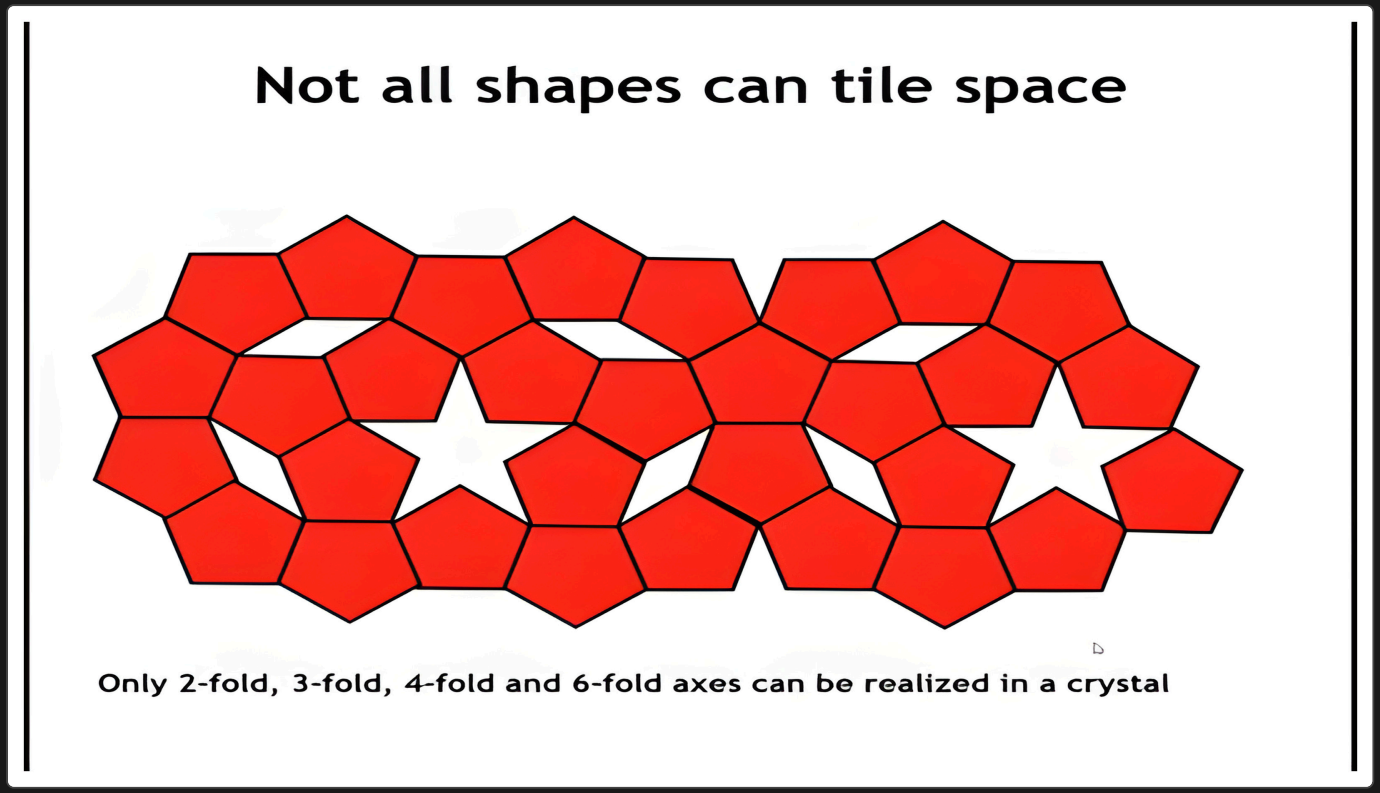


- $\mathbf{a}$ and $\mathbf{b}$ are the lattice vectors, every lattice point can be defined by adding these two vectors (in 2D)
- The unit cell is the **parallelogram** defined by the lattice vectors, all space can be filled by tiling unit cells.

WHICH SHAPES CAN WE USE TO TILE SPACE?

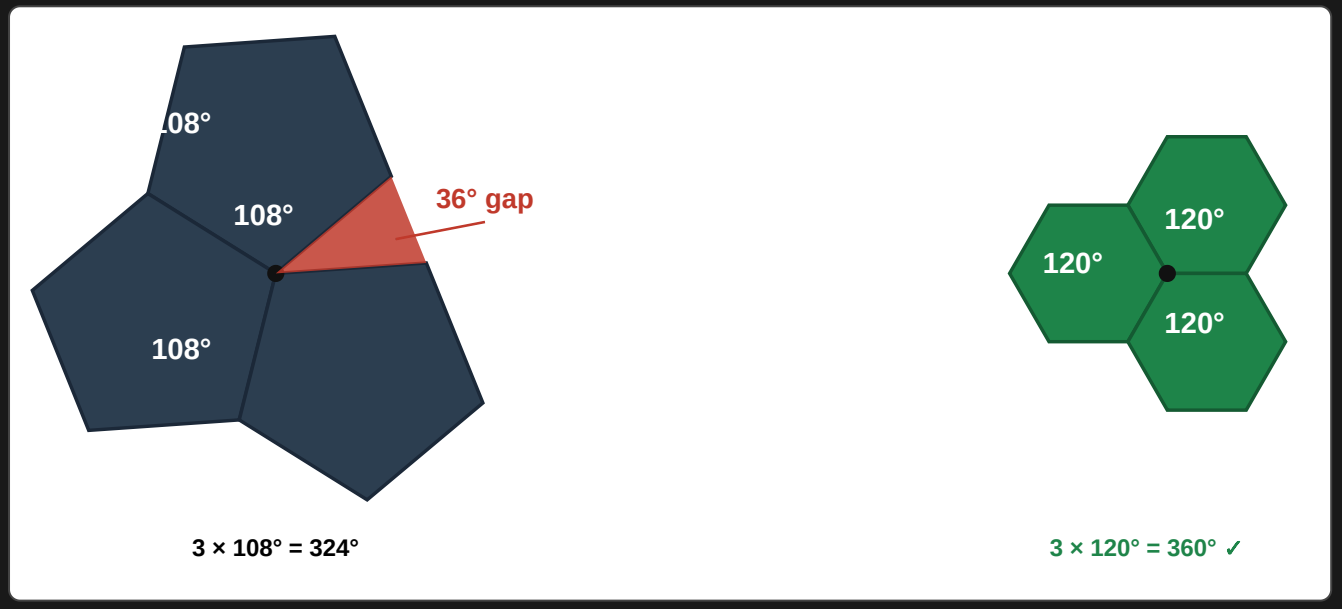


NOT ALL SHAPES CAN TILE SPACE



Regular pentagons leave gaps however they are packed.

WHY FIVE-FOLD CAN'T TILE: THE 360° TEST



The rotation rule

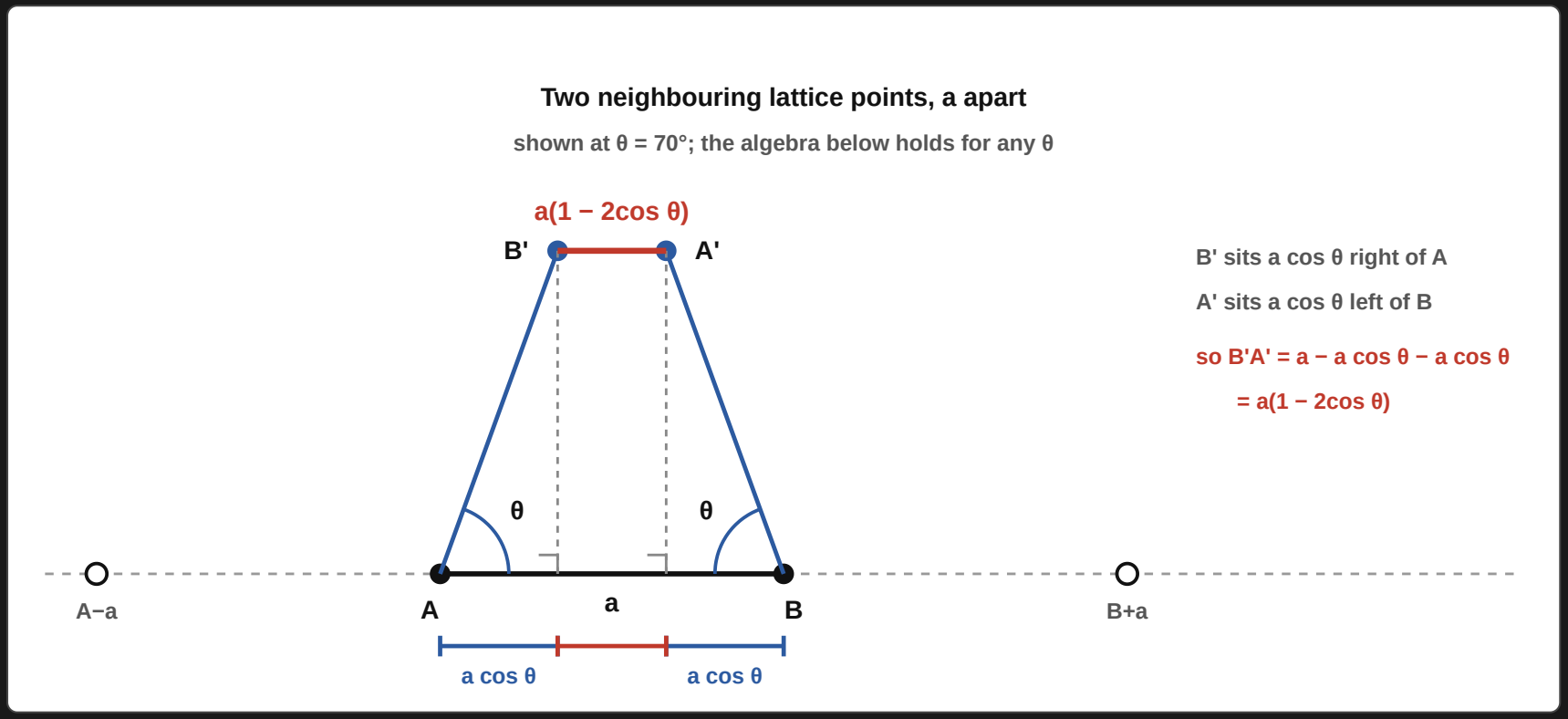
A lattice can repeat after a turn by θ **only if** $2\cos\theta$ is a whole number.

θ	fold	$2\cos\theta$
60°	6 ✓	1
90°	4 ✓	0
120°	3 ✓	−1
180°	2 ✓	−2
72°	5 ✗	0.618
45°	8 ✗	1.41

72° (5-fold) and 45° (8-fold) don't give whole numbers, so no lattice looks the same after those turns — the pentagon *and* the octagon fail.

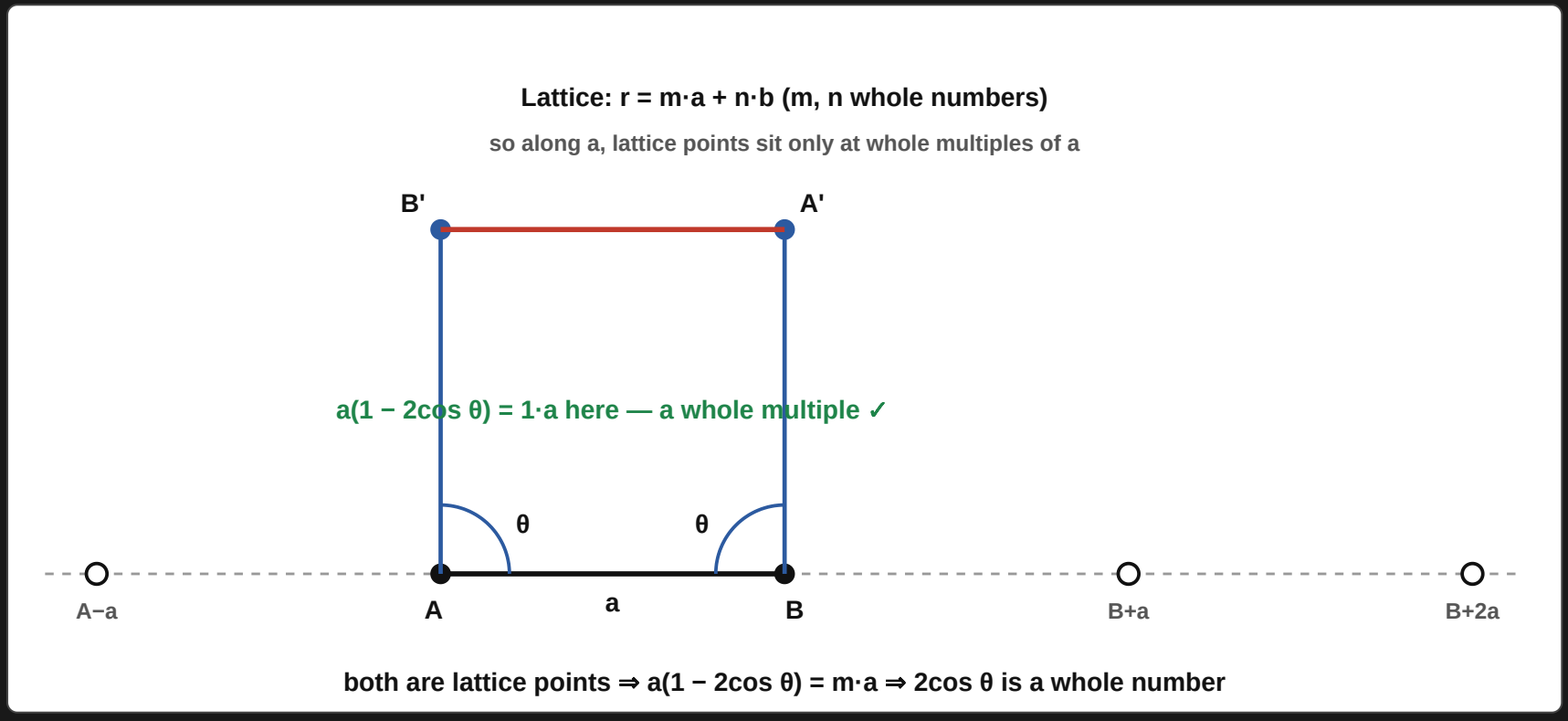
Copies meeting at a point must sum to exactly 360°. Pentagons (108°) give 3×108=324° → a 36° gap; hexagons (120°) give 3×120=360° → perfect. Per *Solid State Materials Chemistry*, §1.1.3.

WHERE $A(1 - 2\cos\Theta)$ COMES FROM



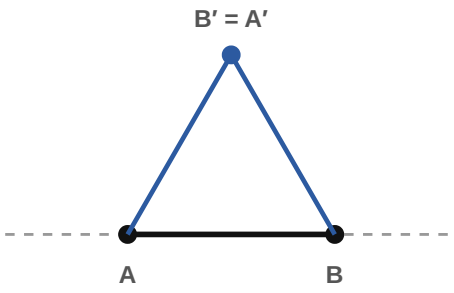
Each rotation moves its endpoint $a\cos\theta$ along a, so two rotations shorten the gap by $2a\cos\theta$. This is the length; which θ a lattice allows is the next slide.

THE PROOF — PART 1: WHAT A LATTICE ALLOWS

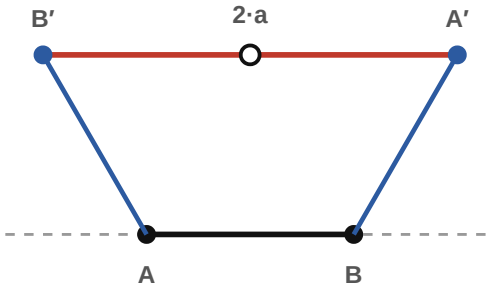


By definition a lattice is the points $m \cdot a + n \cdot b$ with whole m, n — so along a, separations come only in whole multiples of a.

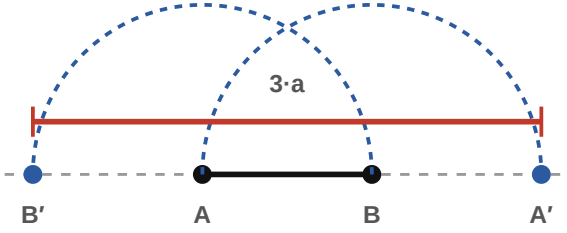
THE OTHER ALLOWED TURNS: 60°, 120°, 180°



θ = 60° → 0·a ✓



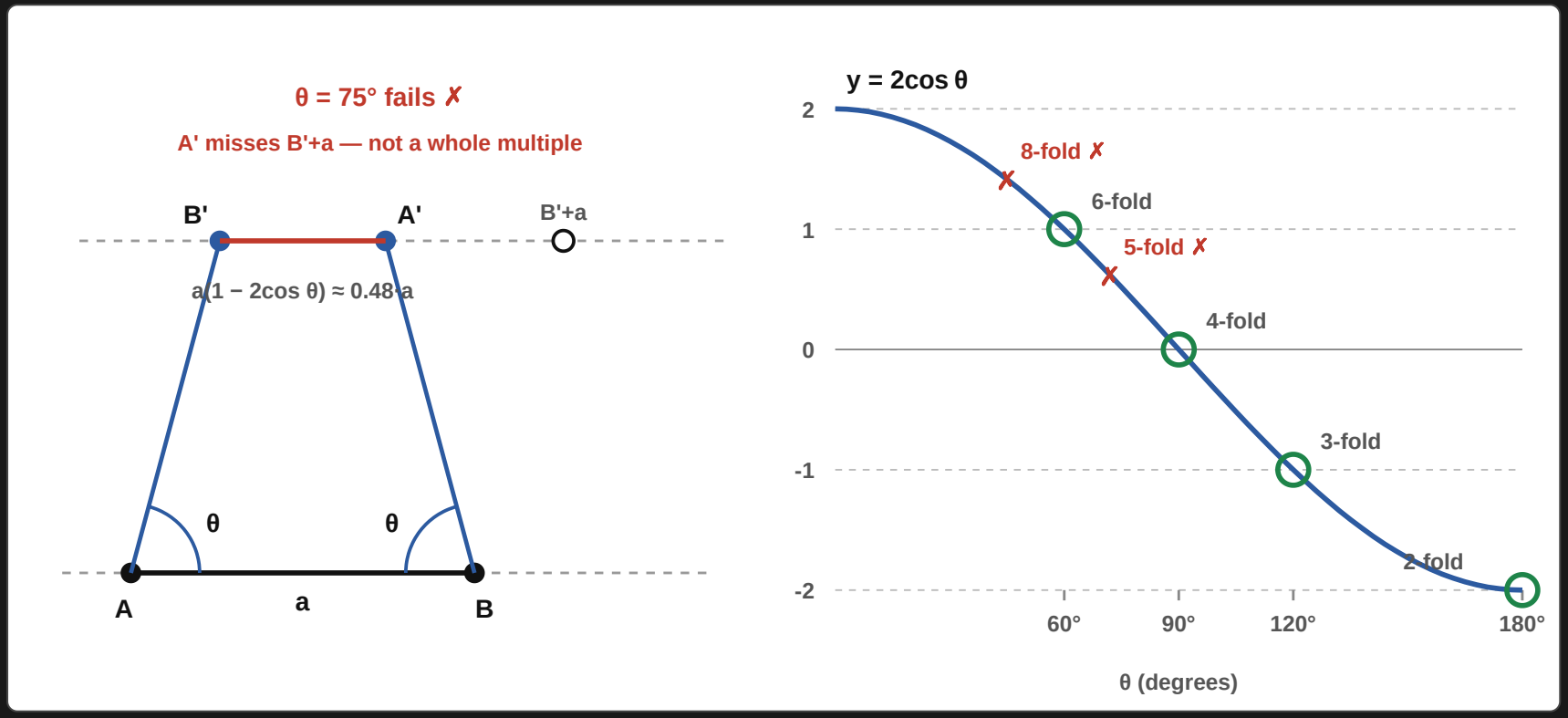
θ = 120° → 2·a ✓



θ = 180° → 3·a ✓

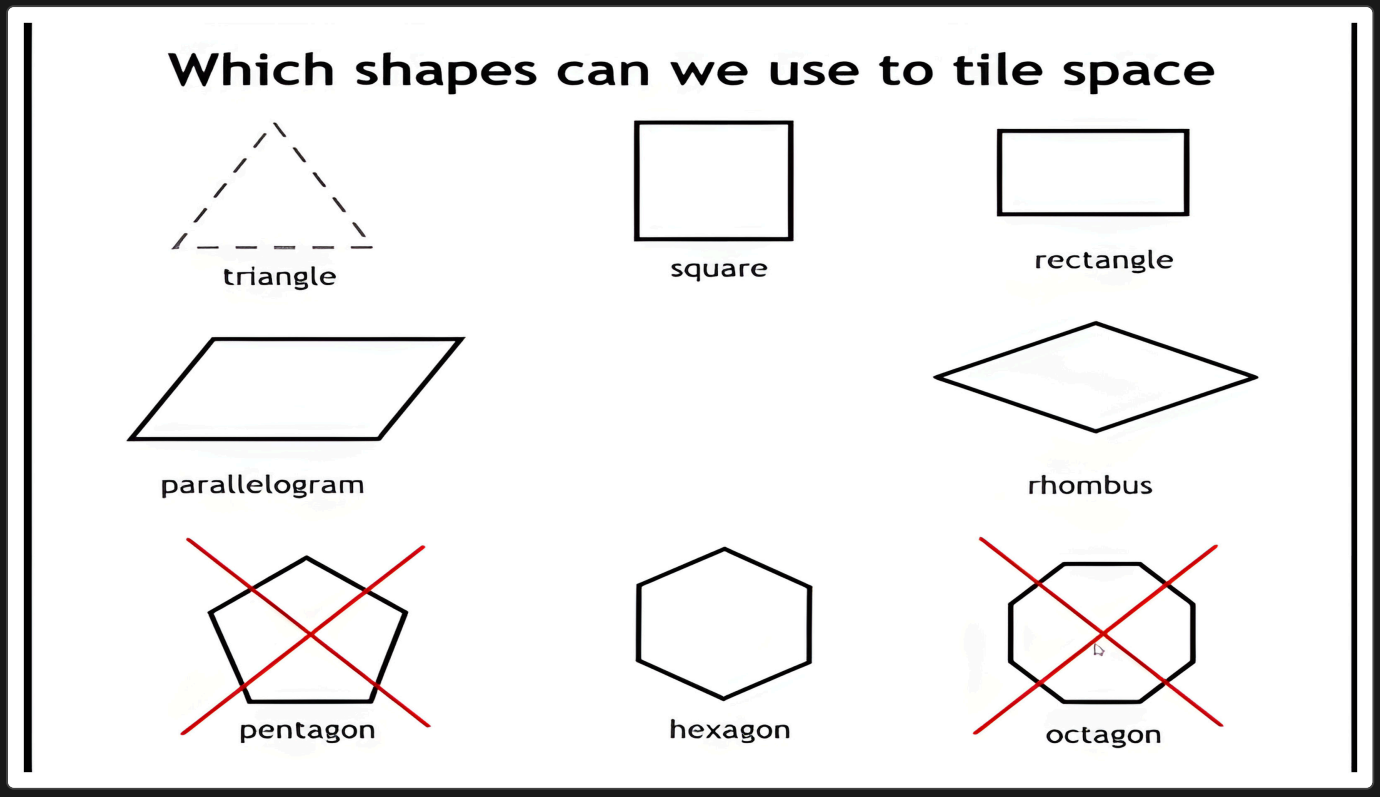
Separations: 60° → 0·*a*, 90° → 1·*a*, 120° → 2·*a*, 180° → 3·*a* — whole multiples, all four.

THE PROOF — PART 2: ONLY FOUR ANGLES PASS



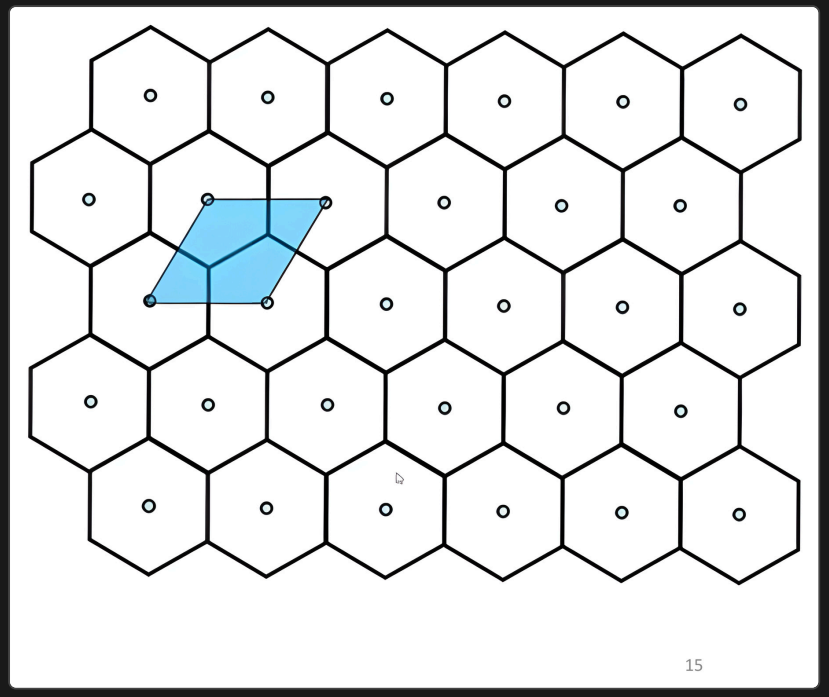
At θ = 75° the separation is 0.48·*a* — not a whole multiple, so no lattice has a 75° axis. Only θ = 180°, 120°, 90°, 60° make 2cos θ whole.

WHICH SHAPES TILE THE PLANE?



Which of the eight cannot tile the plane alone? **Verdict:** the pentagon and octagon fail; the triangle needs rotation; the rest tile by translation.

FROM A HEXAGON TILING TO A LATTICE

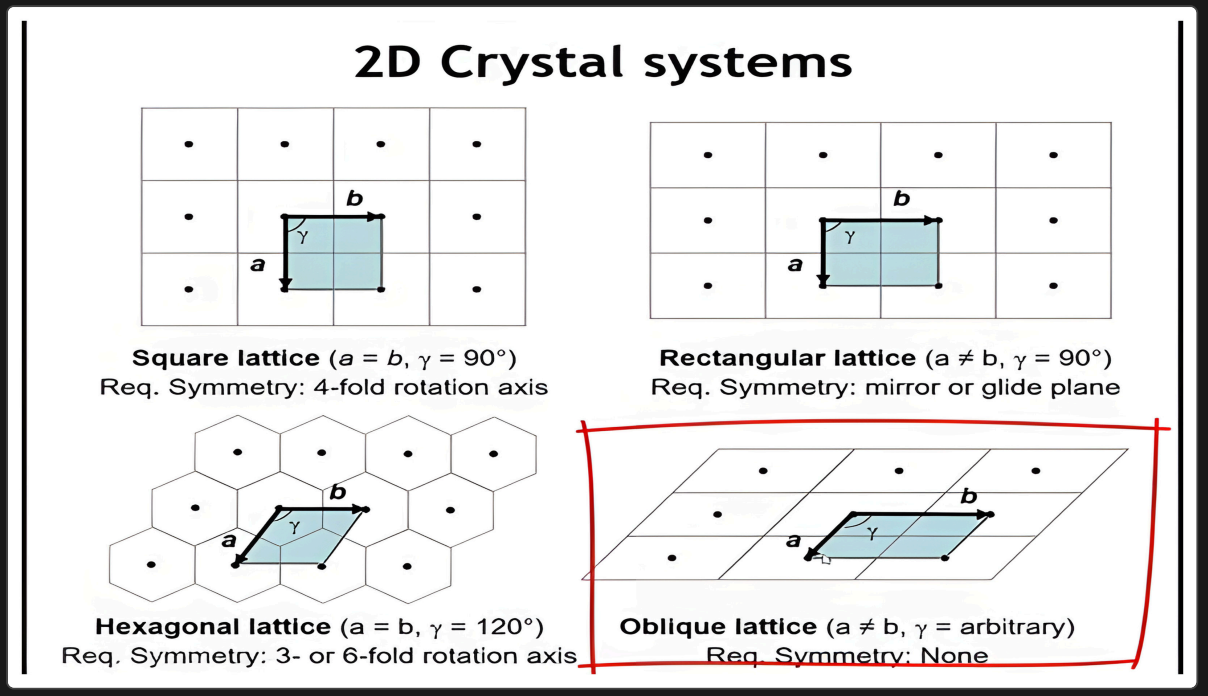


Two translation vectors span a parallelogram, never a hexagon. Where is the lattice?

- Mark one **equivalent point** per repeat: the hexagon centers form the **lattice**.
- Neighboring centers span the **unit cell**: a rhombus, $|\mathbf{a}| = |\mathbf{b}|$, angles $60^\circ/120^\circ$.

"Hexagonal" names the points' six-fold symmetry (six neighbors, 60° apart), not the cell shape; each point's territory is one hexagon.

2D CRYSTAL SYSTEMS

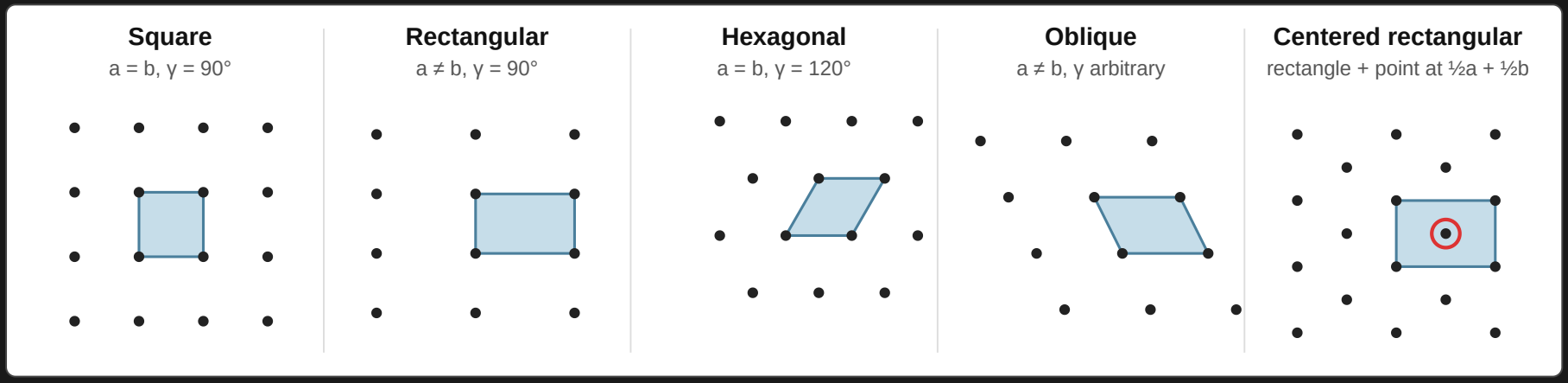


A crystal system groups patterns by their rotational symmetry: the relations between a, b, γ decide which rotations and mirrors are possible.

- **Square**: $a = b,\ \gamma = 90^\circ$ → 4-fold axis.
- **Rectangular**: $a \neq b,\ \gamma = 90^\circ$ → mirror / glide.
- **Hexagonal**: $a = b,\ \gamma = 120^\circ$ → 3- or 6-fold.
- **Oblique**: $a \neq b,\ \gamma$ arbitrary → none.

2D BRAVAIS LATTICES

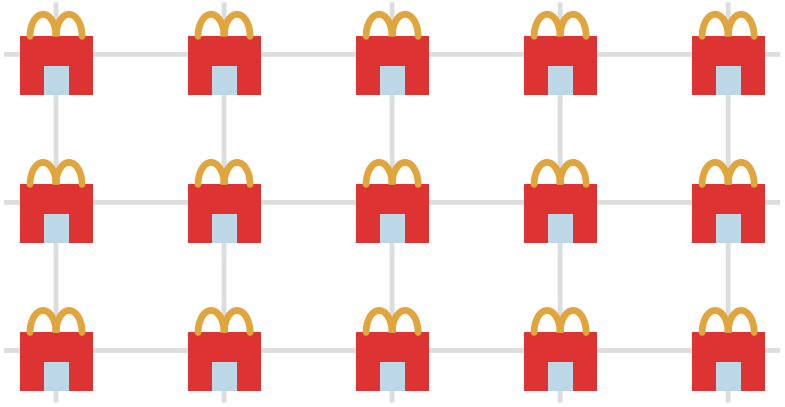
A Bravais lattice is the pattern of equivalent points before any atoms are placed: every point has the same surroundings.



Exactly five distinct 2D lattices exist: four primitive, plus centered rectangular. Per *Solid State Materials Chemistry*, Problem 1.2 (§1.6) and its solutions-manual answer.

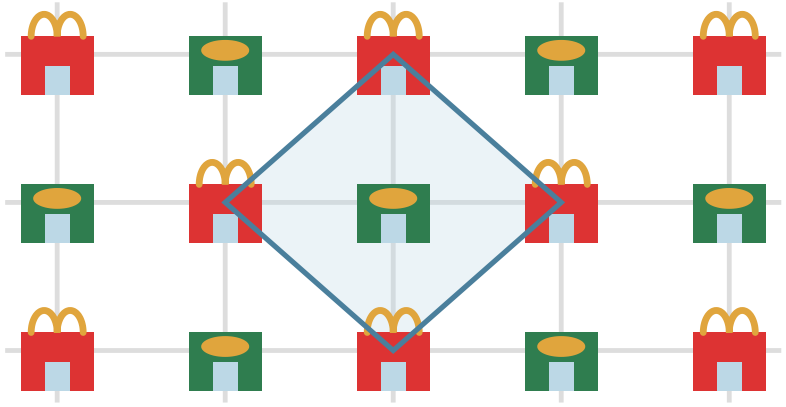
WHICH POINTS COUNT AS LATTICE POINTS?

This works



Every corner is the same McDonald's, facing the same way:
every restaurant is a lattice point.

This doesn't



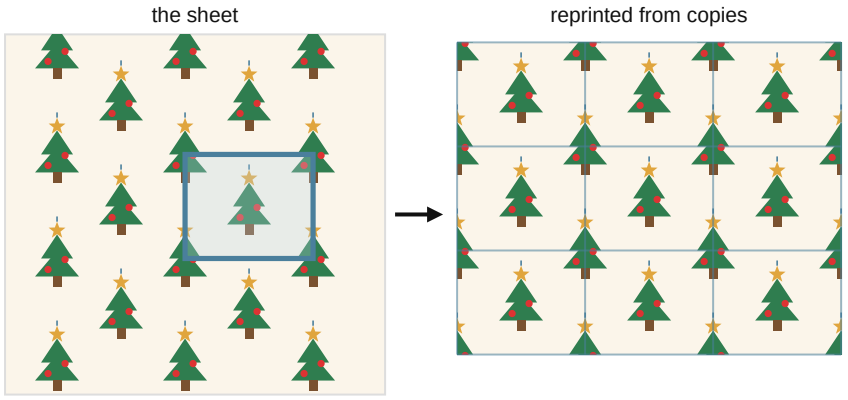
Alternate corners are a Subway — not the same point.
The true repeat is the bigger, tilted McDonald's-to-McDonald's cell.

A lattice point must look identical — same thing, same surroundings, same orientation.

Per *Solid State Materials Chemistry*, §1.1.4 (Bravais Lattices), p. 5.

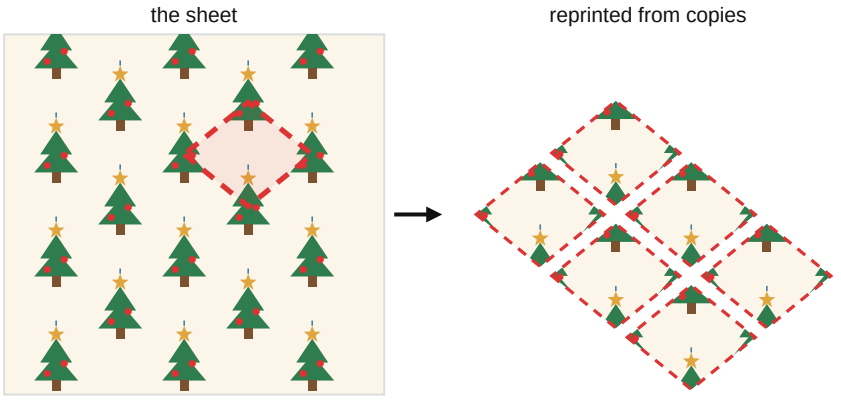
CHOOSING A CELL: CROP THE PIECE THAT SHOWS THE PATTERN

This works



Each rectangular copy shows a whole tree and its mirror;
the copies stack straight back into the sheet.

This doesn't



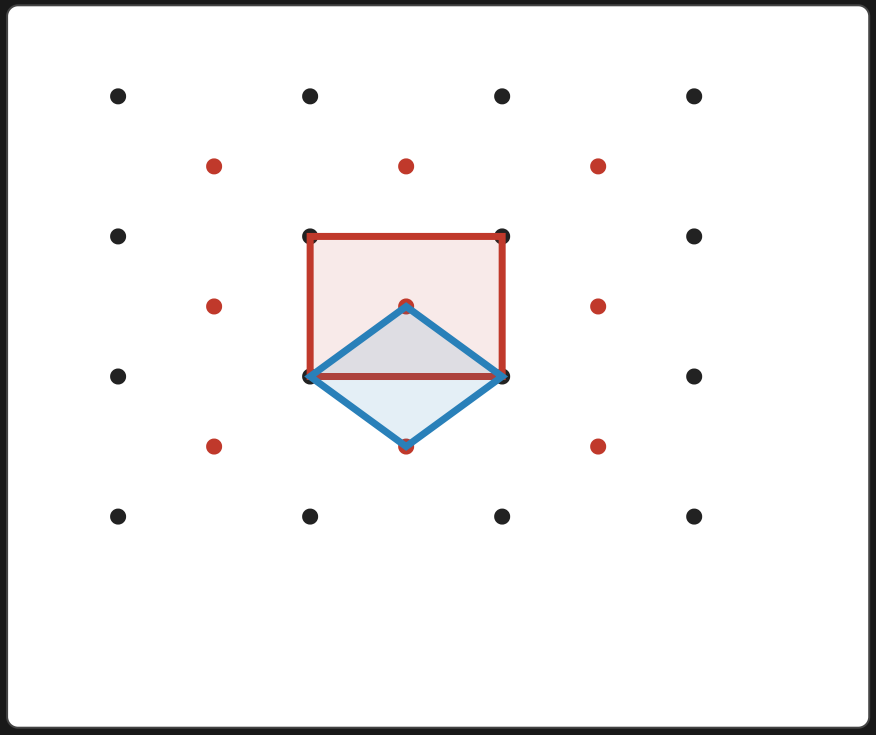
Half the paper per copy — but every edge slices through trees:
no copy shows the pattern, and the reprint is a jigsaw.

We choose the cell that shows the symmetry, even when a smaller one exists.

Per *Solid State Materials Chemistry*, Problem 1.2 (§1.6) and its solutions-manual answer.

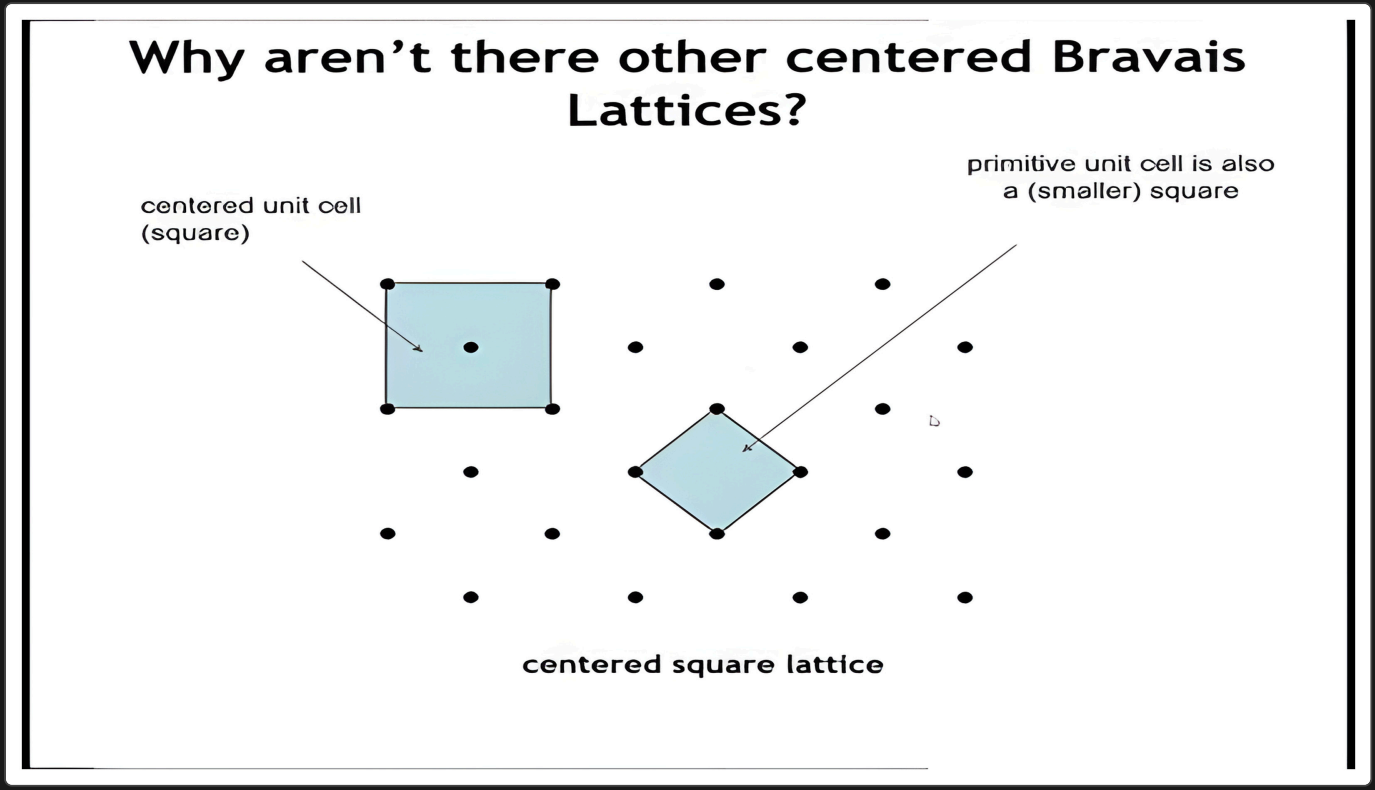
WHY KEEP "CENTERED RECTANGULAR"?

Same point set, two cell choices. Toggle to see why the rectangle is preferred over the smaller oblique cell.



The rectangular cell exposes the lattice's mirror symmetry that the smaller oblique primitive cell hides. Per *Solid State Materials Chemistry*, Problem 1.2 (§1.6) and its solutions-manual answer.

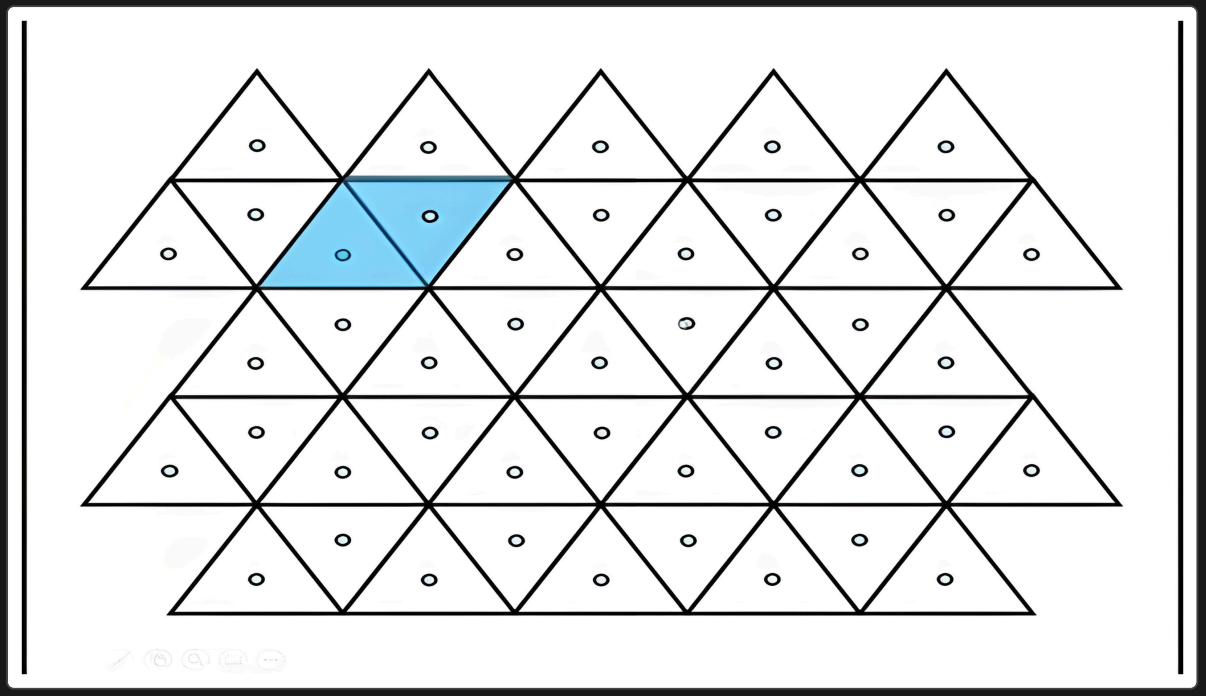
WHY AREN'T THERE OTHER CENTERED BRAVAIS LATTICES?



- A "centered square" is **not** new — a smaller primitive square, rotated 45° with half the area, generates the same points.

Drawing the diagonals reveals the smaller primitive square.

TRIANGLES TILE TOO — SAME HEXAGONAL LATTICE

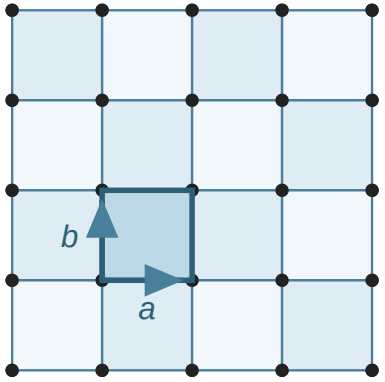


- "Up" triangles alone leave gaps; you also need "down" triangles.
- Mark the triangle centers → each point has 6 neighbors at 60°.
- Unit cell = rhombus, $|\mathbf{a}| = |\mathbf{b}|, \gamma = 60^\circ$ → same hexagonal Bravais lattice.

WHAT COUNTS AS A UNIT CELL?

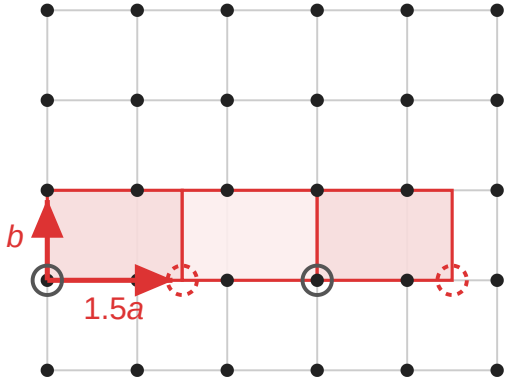
A unit cell is a parallelogram built on two lattice vectors: copied along its own two edges it tiles the plane, and every copy contains the lattice points identically.

A cell on *a* and *b*



✓ Every copy is alike, with no gaps and no overlaps. Points per cell is the same number in every copy.

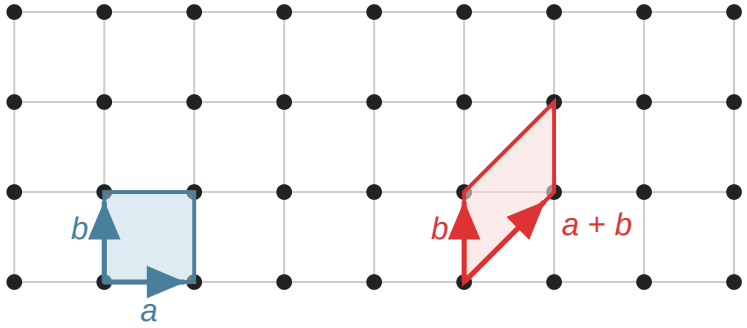
A candidate with edges 1.5*a* and *b*



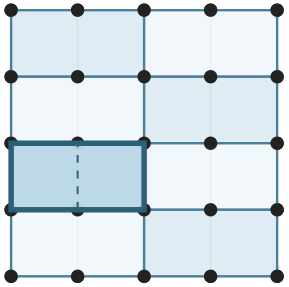
✗ 1.5*a* is not a lattice vector: the copy corners land on a point, then between points, alternating; the points sit in different places inside neighboring copies. A shape whose copies differ is not a unit cell.

A shape is a unit cell exactly when its two edges are lattice vectors; only then do its copies tile the plane with every copy holding the lattice points the same way.

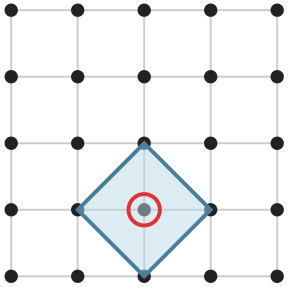
SAME LATTICE, MANY CELLS



one point — one point, same area



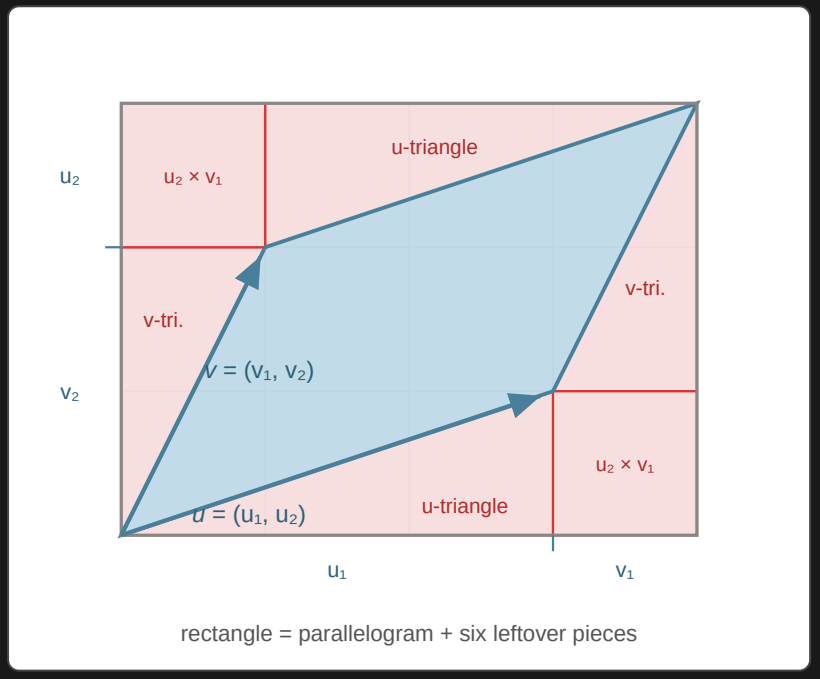
$(2a, b)$
2 squares, 2 points
4 × ¼ corners + 2 × ½ edges
8 copies, 16 squares, 16 points



$(a + b, a - b)$
2 squares, 2 points
4 × ¼ corners + 1 inside

The copies tile with no gaps and every copy is alike, and the lattice carries one point per original square, so a cell covering k squares holds k lattice points. To count them: a corner point is shared by four cells and counts $\frac{1}{4}$; an edge point is shared by two and counts $\frac{1}{2}$; a point inside counts 1.

WHY THE CROSS-MULTIPLIED DIFFERENCE IS THE AREA



rectangle = parallelogram + six leftover pieces

rectangle

$$(u_1 + v_1)(u_2 + v_2) = u_1u_2 + u_1v_2 + u_2v_1 + v_1v_2$$

minus the two u-triangles

$$2 \times \tfrac{1}{2}\, u_1u_2 = u_1u_2$$

minus the two v-triangles

$$2 \times \tfrac{1}{2}\, v_1v_2 = v_1v_2$$

minus the two corner rectangles

$$2\, u_2v_1$$

parallelogram

$$u_1v_2 + u_2v_1 - 2\, u_2v_1 = u_1v_2 - u_2v_1$$

For cell edges $m_1\mathbf{a} + n_1\mathbf{b}$ and $m_2\mathbf{a} + n_2\mathbf{b}$, the same picture gives the area $|m_1n_2 - m_2n_1|$, counted in original squares.

THE SHORTCUT: JUDGE ANY CELL FROM FOUR NUMBERS

Write each edge on the old ones, with whole-number coefficients:

$$\mathbf{a}' = m_1\mathbf{a} + n_1\mathbf{b}$$
$$\mathbf{b}' = m_2\mathbf{a} + n_2\mathbf{b}$$

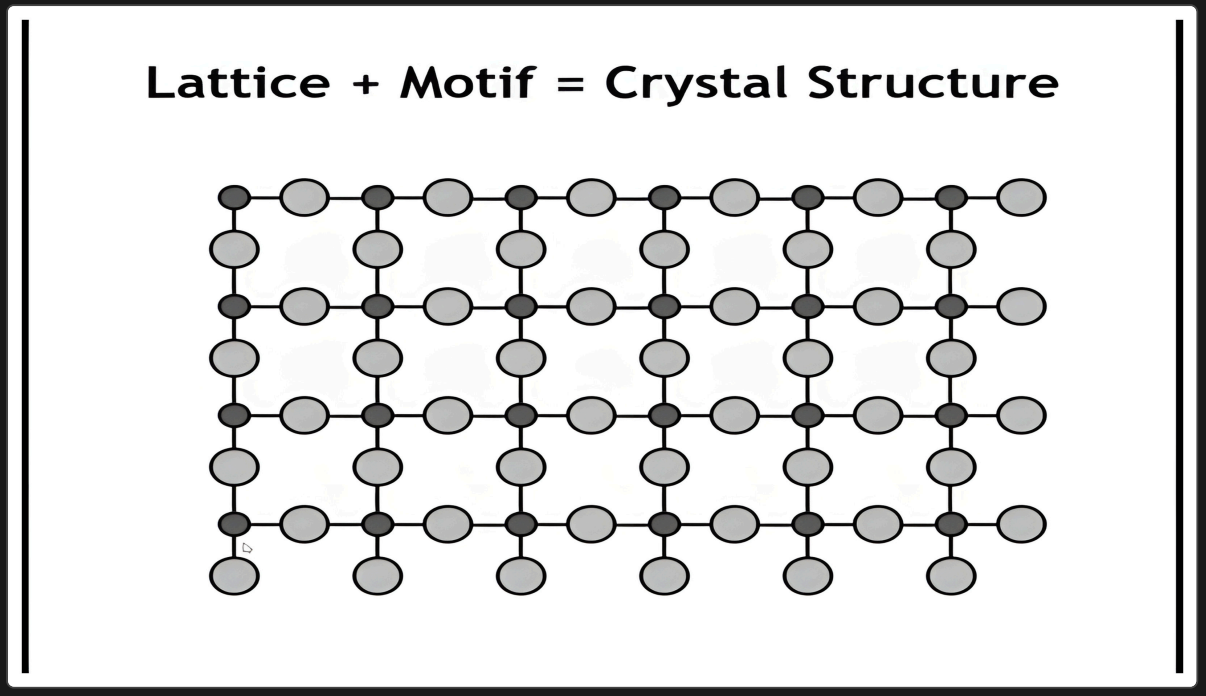
The count: the cell covers $k = |m_1 n_2 - m_2 n_1|$ original squares and holds k lattice points.

The verdict: whole numbers with $k \geq 1$ make a genuine cell of k points, primitive exactly when $k = 1$. An edge with a coefficient that is not whole is not a lattice vector — no cell, no test. $k = 0$ means the edges are parallel — no parallelogram, no cell.

Cell	Coefficients	Cross-multiplied difference	Points	Verdict
$(2\mathbf{a},\ \mathbf{b})$	$(2,0),\ (0,1)$	$\lvert 2\cdot 1 - 0\cdot 0\rvert = 2$	2	not primitive
$(\mathbf{a}+\mathbf{b},\ \mathbf{a}-\mathbf{b})$	$(1,1),\ (1,-1)$	$\lvert 1\cdot(-1) - 1\cdot 1\rvert = 2$	2	not primitive
$(\mathbf{a}+\mathbf{b},\ \mathbf{b})$	$(1,1),\ (0,1)$	$\lvert 1\cdot 1 - 0\cdot 1\rvert = 1$	1	primitive
$(1.5\mathbf{a},\ \mathbf{b})$	$(1.5,0),\ (0,1)$	1.5 is not whole — no test	—	not a cell
$(2\mathbf{a}+\mathbf{b},\ 4\mathbf{a}+2\mathbf{b})$	$(2,1),\ (4,2)$	$\lvert 2\cdot 2 - 4\cdot 1\rvert = 0$	0	not a cell

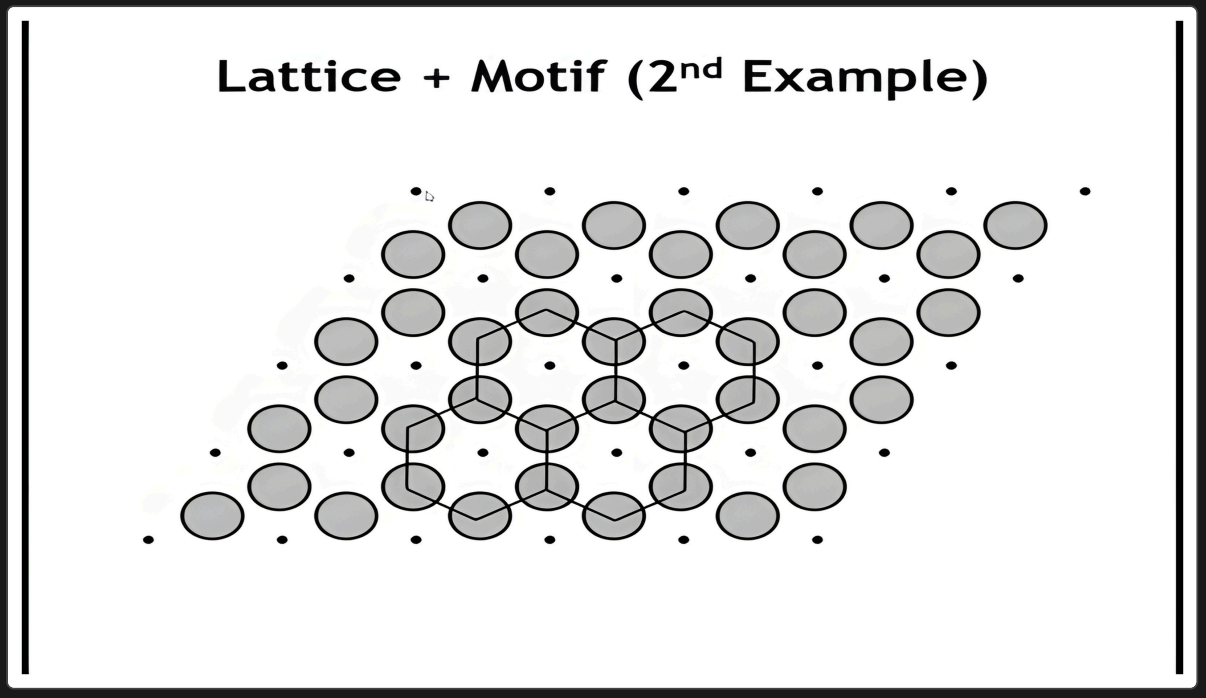
Four whole-number coefficients decide everything: cell or not, k points, primitive exactly when $k = 1$. k is the cell's area in primitive-cell units — *Solid State Materials Chemistry* §1.1.4 counts the hexagonal setting of R this way, three lattice points for triple the volume, and the same count makes C and I two-point cells and F a four-point cell, kept for symmetry.

LATTICE + MOTIF = CRYSTAL STRUCTURE



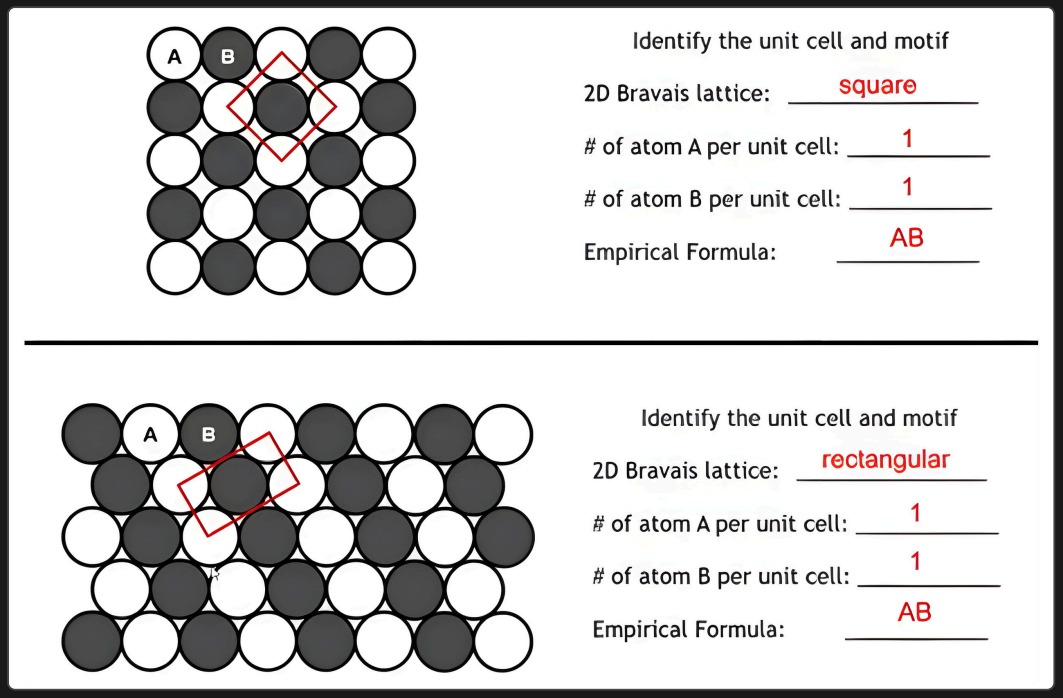
- Place the $\mathrm{CuO_2}$ motif on **one** lattice point (fixed orientation).
- Repeat the same motif on **every** lattice point.
- Draw bonds → a $\mathrm{CuO_2}$ **square-net** layer (the cuprate plane).

LATTICE + MOTIF (2ND EXAMPLE): GRAPHENE



- Motif = **2 carbons** at fixed offsets from a lattice point (not on it).
- Repeat across the hexagonal lattice → carbons line up.
- Each C is 3-coordinate at 120° → the **graphene** honeycomb.

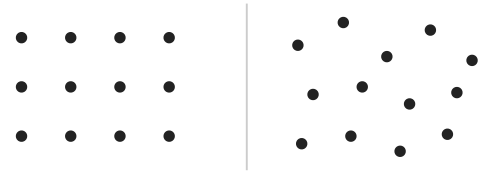
PRACTICE: IDENTIFY THE UNIT CELL & MOTIF



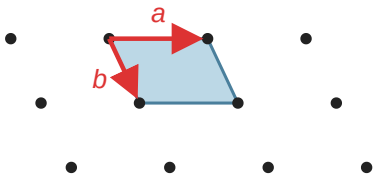
- **Top**: square lattice, A at corners + 1 B at center → $4 \times \frac{1}{4}$ A $= 1$ A, 1 B → **AB** (square Bravais, 1-atom motif).
- **Bottom**: B at center is a *different* atom → **not** centered-rectangular; it is primitive rectangular with a 2-atom motif → **AB**.

SUMMARY — LECTURE 1: TRANSLATIONAL SYMMETRY IN 2D

Crystalline vs. amorphous



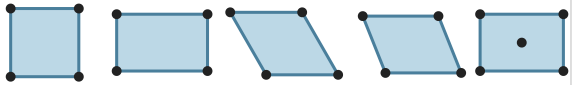
Lattice + unit cell



Allowed rotations: 2, 3, 4, 6



Five 2D Bravais lattices



Lattice + motif = crystal



CuO_2 plane · graphene



TRANSLATIONAL SYMMETRY IN 3D

Goal

Introduce the **3D unit cell** and the classification of **Bravais lattices** as the foundation for describing all crystal structures.

Learning Objectives

- Recognize the **7 lattice systems** and their defining features
- Distinguish **primitive**, **body-**, **face-**, and **base-centered** lattices
- Understand how **14 Bravais lattices** arise from translational symmetry

Based on *Solid State Materials Chemistry*, Ch. 1 §1.1 (pp. 1–12)

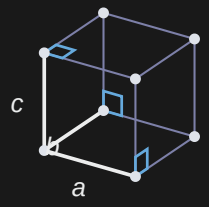
THE 3D UNIT CELL

A **unit cell** is the smallest repeating **volume** that can be translated through space (by lattice vectors) to build the entire crystal lattice.

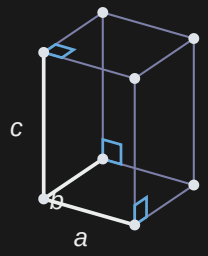
- In **2D**, the unit cell is an area (a parallelogram).
- In **3D**, it's a volume (a **parallelepiped** — six faces, opposite faces parallel, each face a parallelogram).

THE SEVEN LATTICE SYSTEMS

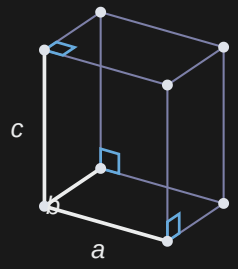
Seven shapes a 3D unit cell can take. Crystals get their own seven-way grouping — the crystal systems — once we can talk about rotations.



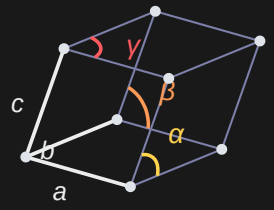
Cubic
a = b = c
α = β = γ = 90°



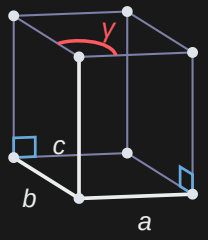
Tetragonal
a = b ≠ c
α = β = γ = 90°



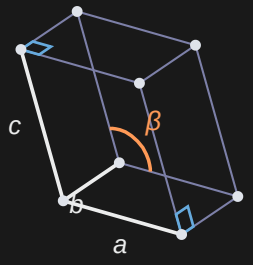
Orthorhombic
a ≠ b ≠ c
α = β = γ = 90°



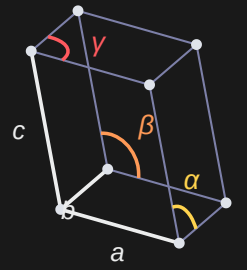
Rhombohedral
a = b = c
α = β = γ ≠ 90°



Hexagonal
a = b ≠ c
α = β = 90°, γ = 120°



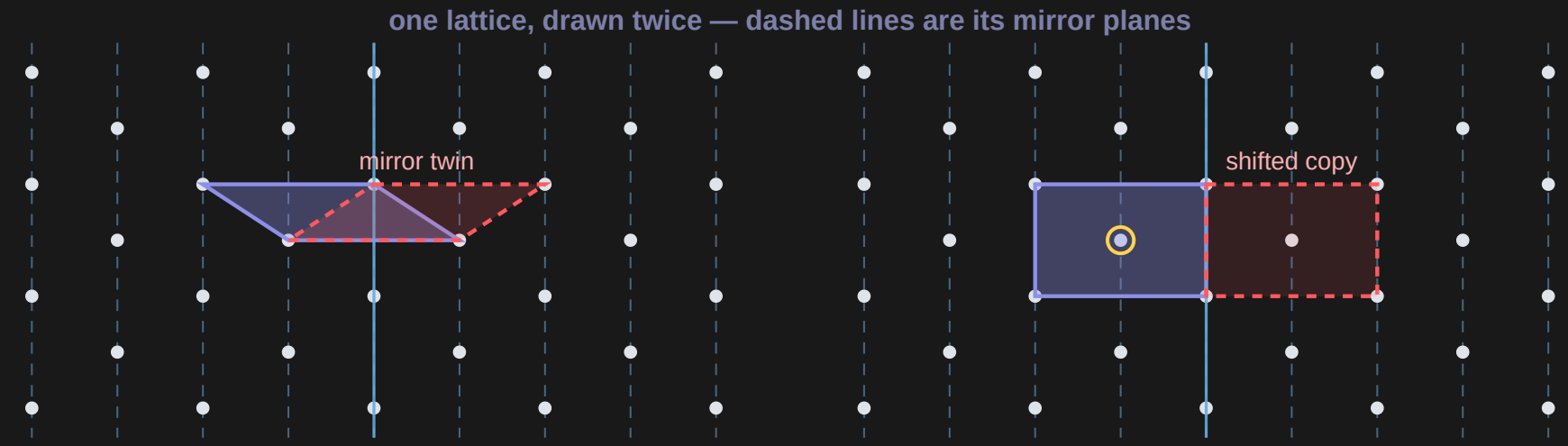
Monoclinic
a ≠ b ≠ c
α = γ = 90°, β ≠ 90°
b ⊥ a, c



Triclinic
a ≠ b ≠ c
α ≠ β ≠ γ ≠ 90°

Cyan squares: angles forced to 90°. Colored arcs: the free angles each system leaves open.

ONE LATTICE, TWO CELLS



1 lattice point — smallest cell, slanted, askew to the mirrors

2 lattice points — bigger cell, mirrors run along its edges

The rule: choose a cell that **keeps the full symmetry** first, the **smallest** such cell second. Reflected, the rectangle lands on a shifted copy of itself; the parallelogram lands on its red mirror twin, and no shift lays the twin back on the drawn cell.

LATTICE CENTERING

Lattice centering tells us how many **lattice points** are inside a chosen **unit cell** (beyond just the corners).

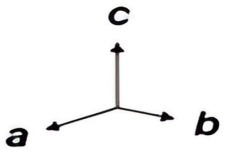
- A **primitive (P) unit cell** has **only corner lattice points**, which add up to exactly 1 lattice point per cell.
- A **centered unit cell** has additional lattice points located inside the cell (body center or face centers).

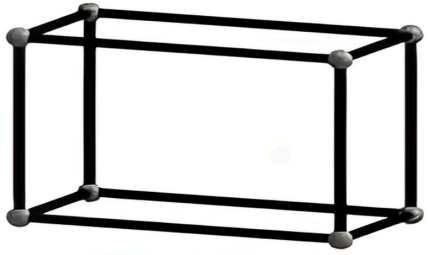
The interior points are **ordinary points of the same lattice** — the gold-circled point of the last slide. A symmetry-first cell is bigger than minimal, so some lattice points land inside it; centering names where.

Why centered cells at all? A primitive cell always exists, but its shape can hide the lattice's symmetry; the centered cell keeps the **system's symmetry visible** in its shape.

THE FOUR CENTERING TYPES: P, I, C, F

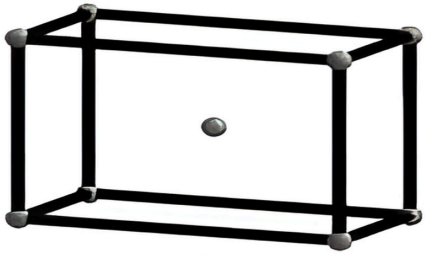
Lattice Centering





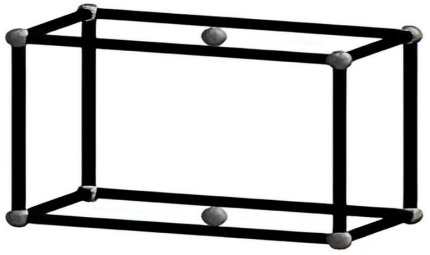
Primitive lattice (P)

no centering



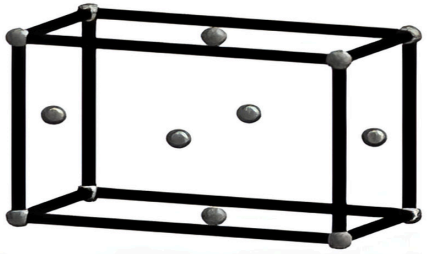
Body centered lattice (I)

*For every atom there is another at ½**a** + ½**b** + ½**c***



Base centered lattice (C)

*For every atom there is another at ½**a** + ½**b***



Face centered lattice (F)

For every atom there are 3 more at:

*½**a** + ½**b***

*½**a** + ½**c***

*½**b** + ½**c***

FROM LATTICE TO BRAVAIS LATTICE

A **Bravais lattice** is the set of points that three primitive **lattice vectors** reach by integer steps:

$$\mathbf{R} = n_1\mathbf{a}_1 + n_2\mathbf{a}_2 + n_3\mathbf{a}_3, \quad n_i \in \mathbb{Z}$$

Three vectors generate every point, so one unit cell carries the whole infinite lattice. A cell has just two properties: its shape and its extra lattice points. The shape is the lattice system; the extra points are the centering.

Next we sort the combinations of system and centering; fourteen distinct ones survive, the fourteen Bravais lattices.

RULES FOR A BRAVAIS LATTICE

The 14 Bravais lattices are the irreducible building blocks of all 3D crystals.

Translational equivalence

Every lattice point must be identical (equivalent environment) when shifted by an integer combination of lattice vectors.

Minimal cell at equal symmetry

Every centered cell contains smaller primitive cells. If one of the primitive cells **keeps the system's full symmetry**, the centering is redundant and thrown out: tetragonal C contains a smaller tetragonal P. If none of them does, the centering is kept: the primitive cells inside cubic F are all rhombohedral, so the F cell is the smallest cell with full cubic symmetry.

Consistency with lattice system symmetry

The centering must keep the symmetry of its lattice system; cubic, for example, requires all edges equal and all angles 90°. A centering that breaks that symmetry is not a distinct lattice of the system.

FROM 28 CANDIDATES TO 14

7 lattice systems × 4 centerings (P, C, I, F) = 28 candidates. Gold dashes: rotation axes. Gold points: the centering.



Triclinic
a ≠ b ≠ c, α ≠ β ≠ γ ≠ 90°
no rotation symmetry



With no symmetry to protect, a smaller slanted cell always exists: every centered candidate is just a bigger drawing of P.



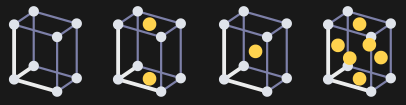
Monoclinic
a ≠ b ≠ c, α = γ = 90°, β ≠ 90°
180° turn about b



The turn pins only the b edge; drawing new a and c edges through the same points turns I and F into C.



Orthorhombic
a ≠ b ≠ c, all angles 90°
180° turns about a, b, c



The three turns force every edge to stay along a, b, c, and the 45° redraw would need a = b: nothing redraws, so P, C, I, F are four different lattices.



Tetragonal
a = b ≠ c, all angles 90°
90° turn about c



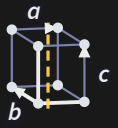
a = b allows the 45° redraw of the square grid: C becomes a smaller P, and F becomes a smaller I.



Cubic
a = b = c, all angles 90°
120° turns about all 4 diagonals



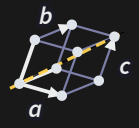
C is broken, not redundant: its face points spoil the diagonal turns, leaving a tetragonal lattice. I and F keep every turn.



Hexagonal
a = b ≠ c, α = β = 90°, γ = 120°
60° turn about c



Any added point either spoils the 60° turn or redraws as a smaller hexagonal P.



Rhombohedral
a = b = c, α = β = γ ≠ 90°
120° turn about the diagonal



Every centered candidate redraws as a smaller rhombohedron. The survivor's letter is **R**, a centering letter it earns from its hexagonal redraw, not from this cell, which has no interior points.

Count the survivors: 1 + 2 + 4 + 2 + 3 + 1 + 1 = **14 Bravais lattices**.

SYSTEMS AND LATTICES ALREADY SPLIT IN 2D

A **crystal system** groups patterns by their rotational symmetry. A **Bravais lattice** is a distinct repeat pattern. 2D has **4 systems** but **5 lattices**.

2D crystal system	Its Bravais lattice(s)
Oblique	oblique
Rectangular	**rectangular P** or **centered rectangular**
Square	square
Hexagonal	hexagonal

One system owning two lattices is not a 3D oddity; you met it in Lecture 1.

In 3D only the trigonal system owns two lattices; the full map is on the next slide.

7 CRYSTAL SYSTEMS, 14 BRAVAIS LATTICES

Crystal system	Defining rotation	Its Bravais lattice(s)
Triclinic	none	P
Monoclinic	one 180° turn	P, C
Orthorhombic	three 180° turns	P, C, I, F
Tetragonal	one 90° turn	P, I
Cubic	four 120° turns	P, I, F
Hexagonal	one 60° turn	**hexagonal P**
Trigonal	one 120° turn	**hexagonal P** or **rhombohedral R**

Hexagonal P serves **two** systems, and trigonal is the only system that picks between **two** lattices. “R-centered hexagonal” (drawn two slides ahead) is the **rhombohedral R** lattice in a hexagonal-shaped cell; it is a different lattice from hexagonal P, and the count stays **14**.

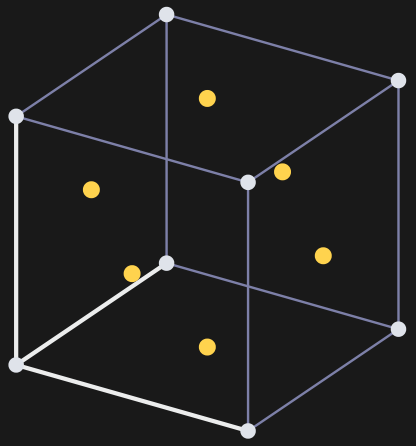
Quartz, $P3_221$ — trigonal, on the hexagonal P lattice

Calcite, $R\bar{3}c$ — trigonal, on the rhombohedral R lattice

Graphite, $P6_3/mmc$ — hexagonal, on the hexagonal P lattice

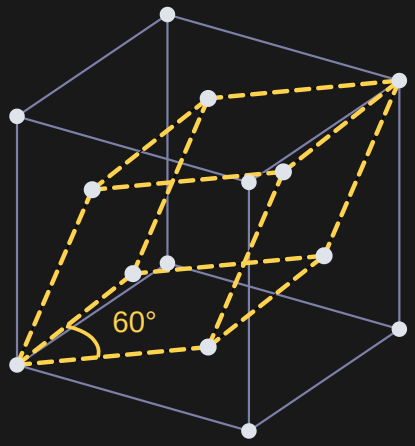
Trigonal names a symmetry, not a cell shape: its crystals turn 3-fold at most and come in two cell shapes, hexagonal P and rhombohedral R. So no trigonal cell appeared among the lattice systems, and there is a rhombohedral **lattice** but no rhombohedral **crystal system**.

ONE LATTICE, TWO CELLS: FCC



Conventional cell — 4 points

8 corners × ⅛ + 6 faces × ½; the cube shows the cubic symmetry

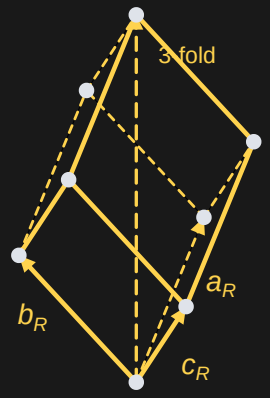


A primitive cell — 1 point

a rhombohedron: a = b = c, all angles 60°
¼ the volume; nothing in its shape says cubic

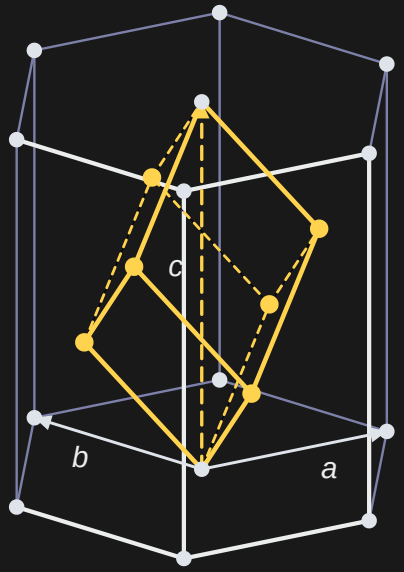
Both drawings hold the **same points**. The cube's shape shows the cubic symmetry; the primitive cell's shape does not. Crystallography chooses the cell that shows the full symmetry first, and the smallest such cell second. That primitive cell is a rhombohedron, and the lattice is still **cubic F**: a rhombohedron-shaped cell does not make a rhombohedral lattice, because symmetry classifies a lattice, not cell shape.

R CENTERING: ONE LATTICE, TWO CELLS



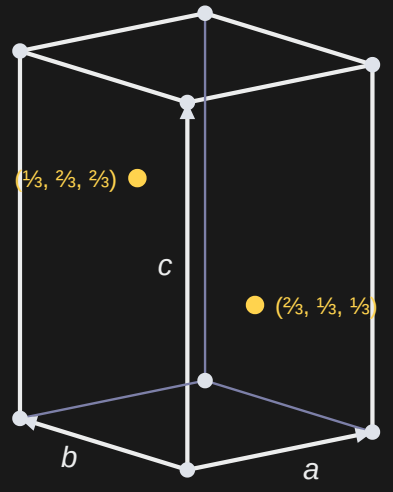
≡

same lattice



⅓

one cell



Primitive rhombohedral

1 lattice point; a = b = c, angles equal, ≠ 90°
3-fold axis down the body diagonal

Hexagonal prism = 3 cells

the rhombohedron's vertices sit on the prism axis and ARE the centering points

R-centered hexagonal — 3 points

a = b, γ = 120°; ⅓ of the prism,
3× the rhombohedron; corners + 2 interior

The gold rhombohedron is the **same cell** in the first two drawings; the third is the conventional **R-centered hexagonal** cell, **⅓ of the prism**. The interior points **break the 6-fold** the empty prism would have — only the 3-fold survives — so both cells describe the **rhombohedral R** lattice from the fourteen, not hexagonal P.

SUMMARY — TRANSLATIONAL SYMMETRY IN 3D

Key points

- A 3D crystal is built by repeating a **unit cell** (a parallelepiped) through translational symmetry.
- **Seven lattice systems**: cubic, tetragonal, orthorhombic, rhombohedral, hexagonal, monoclinic, triclinic.
- **Centering types**: primitive (P), body-centered (I), face-centered (F), base-centered (A, B, C).
- Combining systems + centering gives the **14 Bravais lattices**, the complete set of 3D lattice types.
- This framework underpins all crystallography: every crystal structure belongs to one of these lattices.

HOMEWORK

Work the Lecture 1 practice questions before the next class. They cover the 2D half — lattices and unit cells, the allowed rotations, the five 2D Bravais lattices, and lattice + motif — and the 3D half: the 3D unit cell, the seven lattice systems, lattice centering, and the 14 Bravais lattices.

Open the Lecture 1 practice questions

Every question carries a worked explanation, so you can check your reasoning as you go.